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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXXIV DECEMBER 1931 NUMBER 5

THE PHOTO-ELECTRIC PHOTOMETER OF THE YERKES OBSERVATORY

By JOEL STEBBINS

ABSTRACT

A description of a *photo-electric stellar photometer* designed for use with the 40-inch refractor is given. Among the innovations are a double-slide motion of the photometer for quick setting, a finder with a large field, motor-driven sectors for equalizing the light of stars, filters for measurement of color-indices, and a *Lindemann electrometer*. The photo-electric cell by Kunz is potassium hydride in a quartz tube.

The *sensitivity* of the installation is such that stars as faint as the ninth magnitude can be measured, and the limit of detection in current is about 10^{-15} amperes.

The instrument is being used principally for the determination of *colors of B stars* which are also under spectroscopic observation at the Yerkes Observatory.

In the summer of 1929, I had the privilege of installing and operating a photo-electric photometer on the 40-inch refractor at the Yerkes Observatory. The light-gathering power of the large objective should open a field of work not accessible to smaller instruments, and the particular advantage of the photo-electric cell is that conditions of seeing have no influence on the measures of stars. So long as the light from a star can be passed through a diaphragm a few millimeters in diameter, the cell receiving the extra-focal image will integrate the light-effect and give a photo-electric current proportional to the brightness of the object observed. On this account it was expected that the great telescope could be used on occasions when micrometric or photographic measures would be impossible. On the other hand, the requirements for good and uniform transparency of the atmosphere are just as rigorous for a large instrument as for a small one, and a thick or patchy sky is of no use for photo-

metric work. Fortunately, the surroundings at Williams Bay give no trouble from smoke such as is experienced near towns like Urbana and Madison. Also there are few electric lights in the general neighborhood of Yerkes so that the dark sky background is an advantage in the observation of faint objects.

The addition of this accessory to the equipment of the large telescope presented a number of interesting problems. On the 15-inch refractor at Madison the photometer remains undisturbed at the eye-end for months or even years at a time. For visual observations we simply insert a diagonal eyepiece in front of the focus. On the 40-inch, however, with its varied program by day and night, each change of instruments at the eye-end must be made quickly and conveniently. For this reason the photometer, familiarly known as the "Pep," was designed to be interchangeable with the micrometer and the parallax camera. The photometer weighing 55 pounds (25 kg), together with the battery of dry cells, about 50 pounds (23 kg), are small items to be carried at the end of the large refractor, and when not in use the photometer and battery are hung out of the way on the side of the iron column of the telescope.

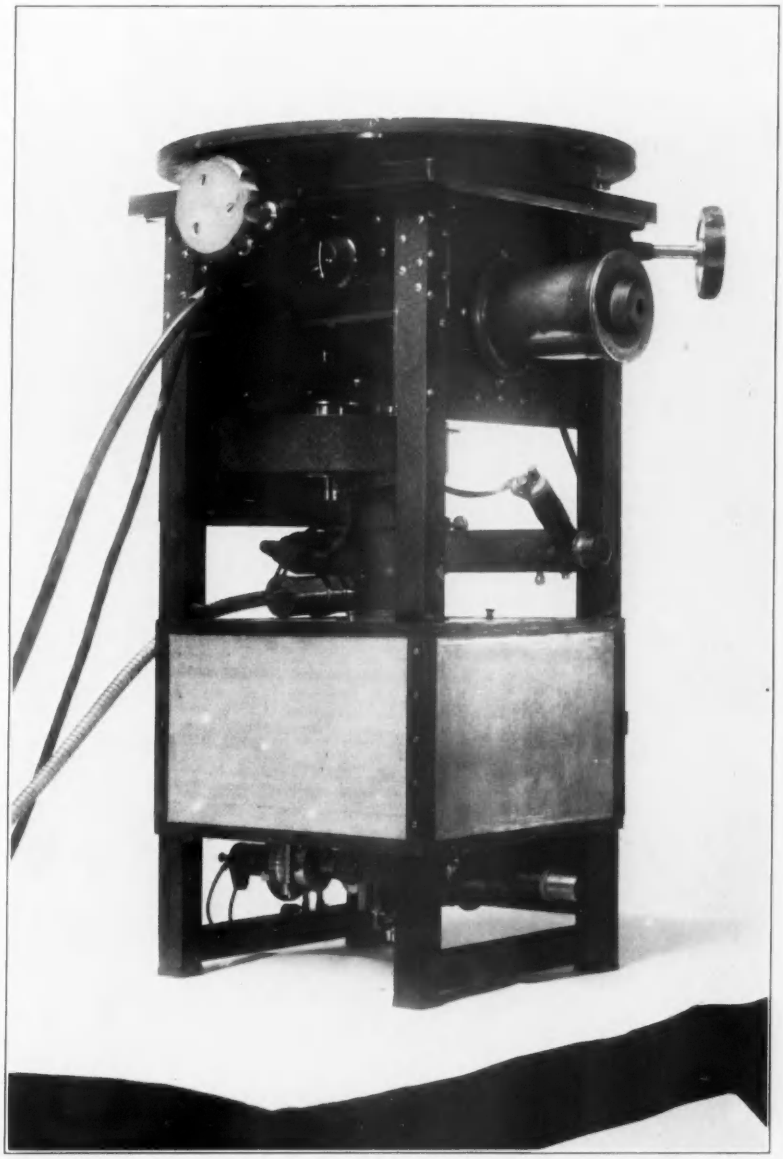
In general, the design of the photometer follows that of the one at Madison,[1] but with the important substitution of the Lindemann electrometer for the swinging string electrometer. The Lindemann electrometer, perfected by the Cambridge Instrument Company, can be tilted at any angle and so makes an ideal instrument for attachment to the end of a moving telescope. We changed to this new form at Madison in 1927, and have found it most convenient in practice.

The instrument was constructed in the shops at the University of Wisconsin under the direction of Mr. O. E. Romare, most of the actual work being done by Mr. Edward Westerberg. Two views of the photometer are shown in Plates XVI and XVII, and the construction of the instrument is perhaps best described by a reference to the function of the different parts.

1. *Attachment to the telescope.*—The photometer is bolted by three capstan screws to the so-called "Adapter A" which is provided for interchange of the photometer, parallax camera, and micrometer.

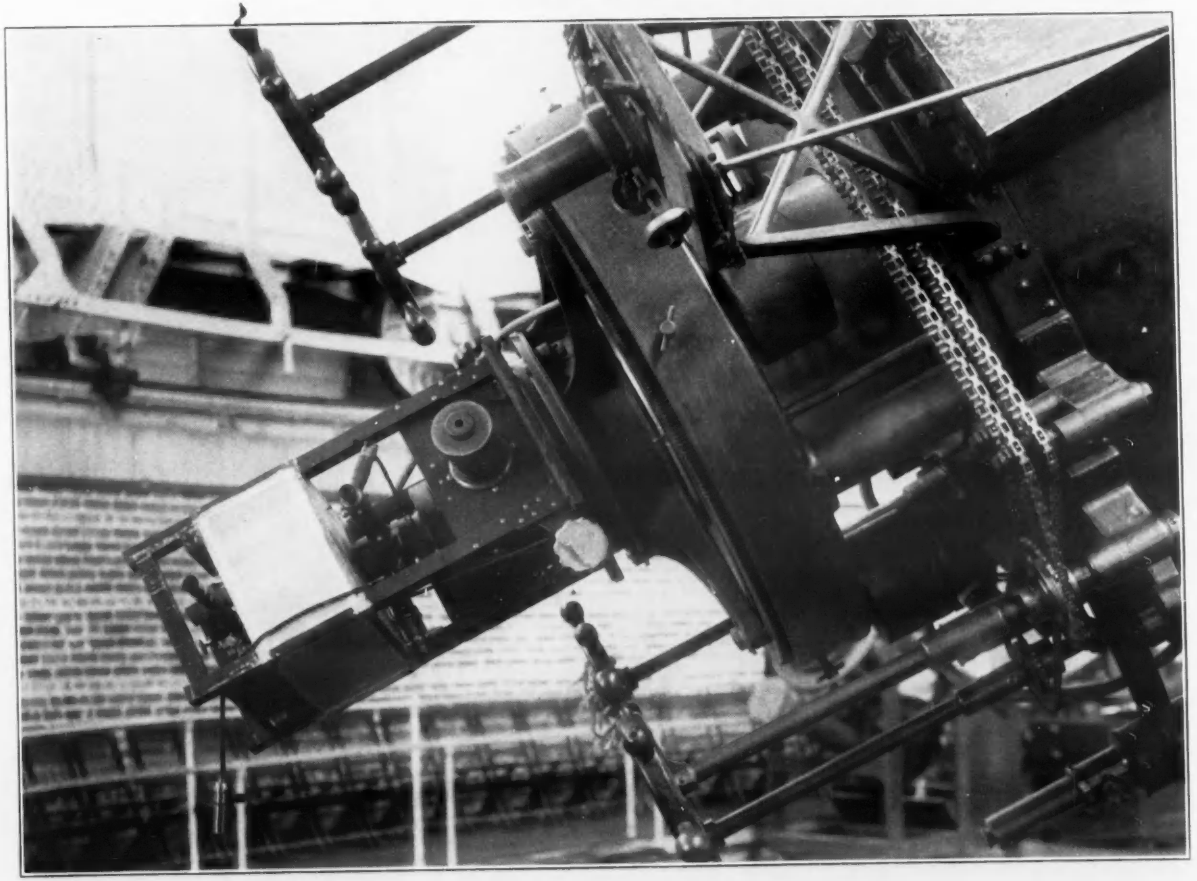
[1] *Publications of the Washburn Observatory*, **15**, 8, 1928.

PLATE XVI



THE PHOTO-ELECTRIC PHOTOMETER. DIAMETER OF UPPER RING, 14½ INCHES

PLATE XVII



THE PHOTOMETER ATTACHED TO THE 40-INCH REFRACTOR

This in turn is fastened to the large ring or tailpiece which can be moved in or out with respect to the main tube. All focusing of the photometer is thus taken care of by the conveniences already supplied at the eye-end.

2. *Rotation in position angle.*—The upper frame of the photometer is attached to a second ring fitting inside of the attaching ring and arranged for rotation of the whole instrument about the optical axis of the telescope. The most convenient position is usually for the observer to be on the side opposite the pier.

3. *Double-slide motion of the whole photometer.*—The upper end of the photometer proper is a square plate rigidly attached to the rotatable ring. This plate supports the slides, one in each co-ordinate, moved by screws, pitch 11 to the inch, turned by conveniently placed handles.

4. *Frame of instrument.*—At the corners of the sliding rectangular plate are attached four square brass tubes of 1-inch size, which make the main frame of the instrument 12 inches square. These tubes are the basis of the remaining construction, and are carried through to the base where they are plugged with soft rubber pieces.

5. *Large diagonal finder.*—Above the primary focus of the 40-inch objective is placed a diagonal finder with a plane mirror which may be turned in or out of the converging beam. This mirror is 3×4.2 inches (75×105 mm), and the field of the large eyepiece is about 2.6 inches (66 mm) or 11 minutes of arc in diameter. Illuminated wires are provided, and when all adjustments have been made this is the only finder that is needed for operation of the photometer. When the telescope is set to 1′ in declination, a moderately bright star can be picked up by sweeping in hour angle without recourse to one of the regular finders on the side of the main tube. The hand slow motions of the 40-inch are used for a closer setting, and the final adjustment is easily made with the double-slide of the photometer. Because of the general slowness of manipulation of so large a telescope it is important that as little time as possible be lost in changing from one object to the next.

6. *Small diagonal finder.*—At the principal focus of the telescope is an iris diaphragm with aperture variable from 3.6 to 34.4 mm, or 38″ to 365″ in arc. Behind this opening, as in my other photometers,

is placed a sliding total-reflecting prism, which will divert the main beam out to another ocular or allow the light to pass on through to the cell. A star's image is refocused by a 25-mm achromatic magnifier onto a pair of illuminated cross wires and viewed with an 18-mm eyepiece. The observer can thus see exactly what light from the source is to pass through to the cell. Ordinarily the large finder is sufficient, but for objects like the components of double stars, etc., the small finder is used.

7. *Occulting sectors.*—For equalizing the light of different objects there is provided a set of five motor-driven rotating sectors mounted permanently on a disk or turret in front of the focal plane. The sectors give equal steps from 0.6 to 3.0 mags., and are arranged so that by rotation of the supporting disk any desired sector is thrown into gear with the motor, and at the position in the optical axis. At intermediate positions the sectors are disconnected and the beam of light is clear. We were put to some trouble to find a motor well enough balanced not to cause vibration of the electrometer needle, and for a time the difficulty seemed insurmountable. Any cushioned mounting on springs or rubber would allow the motor and sectors to wander about in different inclinations of the instrument. The motor finally secured, a 1/200 h-p. from the General Electric Company, satisfies the requirements perfectly, even when attached rigidly to the frame of the photometer. At a motor speed of several hundred revolutions per minute there is not the slightest visible tremor of the electrometer in the microscope.

8. *Color-filters.*—Inclosed in the same box with the sectors is another disk which has four compartments for colored glasses, each of 1-inch opening. By turning an outside arm the observer can bring a desired filter into position, the manipulation being readily accomplished in the dark. The whole motor-sector and color-filter unit is detachable without disturbing the other parts of the instrument.

9. *Cell-box.*—The compartment for holding the photo-electric cell is a box 12 inches (30 cm) square and 6 inches (15 cm) deep. Three of the sides may be removed for convenient access to the interior. A movable frame attached to the top of the box is arranged to slide so as to bring either one of two cells into position, and automatically the proper electrical contact is made. So far, however, only one cell at a time has been used.

10. *Electrometer and reading microscope.*—The Lindemann electrometer is fixed in position below the cell-box, with the terminal for the needle connection brought up inside. An occulting "house" prevents leak of light from the electrometer to the cell. The reading microscope has a double-slide motion; it is fitted with a 16-mm objective, and ocular giving a magnification of about 250. The scale in the eyepiece has 100 divisions, and an average working sensitivity is 50 divisions per volt. At Madison the electrometer is arranged for a double motion and the microscope is fixed. There are advantages in each arrangement; the immovable microscope is less easily displaced by accidental knocks during observing, while it is easier to adapt different shapes of electrometer to a fixed connection with the cell-box. In any case it is necessary to have an adjustment of the zero of the instrument for convenience of observing. The position of the needle could be controlled by a variation of the potential on the plates, but, for routine observations in all positions of the telescope, experience has shown that a mechanical motion of the electrometer or microscope is more reliable than a potentiometer control. When the needle is lost from the field it is a simple matter to take out the eyepiece and pick up the image, moving the electrometer from side to side, without danger of inadvertently throwing the needle over to one of the plates.

11. *Electrical connections.*—The circuits to be used for operation of the photometer are 110 volts direct current for the motor, 6 volts for the small lights, and half a dozen wires for the photometer proper led in conduit from the battery to the cell-box. The lighting circuits are connected by plugs of a different style, while the battery wires are joined by a seven-terminal radio connector. On attaching the photometer to the telescope all circuits are completed in a few seconds simply by connecting in three plugs.

The square tubes making the frame of the instrument form a convenient means for inclosing most of the wiring. At the observer's upper left is a panel with switch and rheostat for controlling the six small lights, one each for the cross-wires of the two finders, one for reading the number of the sector, a test-lamp or artificial star reflected from the back surface of the guiding prism down to the cell, a light for the electrometer, and a general hand-lamp. At the lower right is a second panel with switch and rheostat for the motor.

By mechanical connection of the two toggle switches, the light for locating the sector is turned on only by stopping the motor.

12. *Dry air.*—Instead of depending upon an agent like sodium or phosphorous pentoxide, the cell-box is kept dry by a continuous flow of air bubbled through sulphuric acid. Residual acid fumes are caught in a tube of soda lime, and a bulb of glass wool at the outlet in the box eliminates any dust or bloom from the rubber-tubing. The acid bottles are hung in a frame on the iron column of the telescope, where the compressed air is brought from the thermostat system of the observatory. When not in use the photometer is connected to the air tube, and as the joints of the box are sealed with paraffin there is no perceptible leak of moisture as evidenced by decreased insulation of the cell in the course of a night's work. The electrometer is kept dry with sodium in the side tube furnished for the purpose.

The matter of dry air for the perfect insulation of the system of cell and electrometer is of the utmost importance. It may be that the best solution would be to inclose the terminal of the cell, the grounding key, and the electrometer all in an evacuated space, but, so far, I have not worked out the practical details of such a system. When the photometer is sealed up and left undisturbed, the present arrangement may be satisfactory for weeks or months at a time.

13. *Battery and switches.*—The voltages for the photo-electric cell and the electrometer are supplied by Burgess "B" radio batteries in units of $22\frac{1}{2}$ and $7\frac{1}{2}$ volts up to a total of about 400 volts. These are assembled in a metal-covered box carried on the side of the telescope tube, and easily portable to any place for use with the photometer. The shelf-life of these batteries is about a year, and tests at Madison have shown that the voltage of a unit remains constant within 1 part in 30,000 over periods of a minute or so. We have been unable to trace any of the outstanding errors in the photometric measures to variations in the battery. Some of the units are good for a year and a half or more, so by replacing them at the end of a year there is little trouble from defective cells.

In the battery-box are mounted a series of guard resistances and double-throw switches, one for each connection: to the photo-electric cell, to the standard cell for sensitivity test, to the electrom-

eter plates, and to the ground. The resistances are radio grid-leaks of several megohms each. With all switches thrown to the safety position everything is grounded and there is small danger of accident to the cell or electrometer.

14. *Transfer to 12-inch refractor.*—The design of the attachment ring of the photometer allows it to be bolted to the end of the 12-inch refractor, so that the whole equipment may be carried over and used with that telescope. The battery can be hung on the south side of the column of the 12-inch, and a longer cable of wires carried up over the polar axis and down the side of the tube. So far, however, the photometer has not been used with the smaller telescope.

15. *Photo-electric cell.*—The most important item of the whole installation is, of course, the photo-electric cell. It is now twenty years since my colleague Jakob Kunz at the University of Illinois became interested in the production of alkali cells for astronomical photometry, and it is fifteen years since we made a cell entirely of fused quartz, because of the better insulation of quartz than of glass. Since that time there have been the development of television, the talking pictures, and no end of other applications, so that many companies and firms have become interested in the production of cells and in their improvement. In a way it has been a disappointment to find that no cell commercially available has tested out to be in the same class with the quartz-potassium cells made by Kunz. For instance, a glass cell useful for strong lights has shown a dark current equivalent to the photo-electric effect of a star as bright as Sirius at the focus of the 15-inch refractor. It is the old question of what you want versus what you don't want, and if the cell itself has a current in the dark which is larger than the light-effect to be measured, then no refinement in the installation will overcome this fundamental defect.

16. *Sensitivity of the installation.*—The last decimal place in the figure of merit of a photo-electric cell seems to be more a matter of mystical chance than of any skill or care in preparation. The exact nature of the potassium surface sensitized by a glow discharge in hydrogen is not understood, and two cells prepared side by side as nearly alike as possible may differ considerably in their characteristics. The best cells by Kunz are treasures to be guarded and used

with circumspection, as the difference between a first-class cell and a fair one is as great as the difference between a large telescope and a small one. For instance, at one stage in the installation of the instrument at Yerkes, we were using the best cell out of a half-dozen available, while at the same time another cell on the 15-inch refractor at Madison was giving satisfactory measures of stars fainter than could be reached with the 40-inch with its sixfold greater light-gathering power. At the present writing, however, with a new cell at Yerkes, the two photometers are about equally efficient, and the large telescope has its calculated advantage.

The sensitivity may be represented in figures as follows: At Madison we adopt as a standard star one of spectrum Ao, magnitude 1.0; the equivalent magnitude for the 40-inch would be roughly 3.0. The rate of charge of the electrometer by such an object at the focus of the 40-inch is 2.8 volts per second, or the current is given by the formula

$$I = \frac{C}{9 \times 10^{11}} \cdot \frac{V}{s},$$

where I is the current in amperes, C the capacity of the system in centimeters, and V/s the volts per second. The value of C is approximately 6 cm, so we have 2×10^{-11} amp. for the standard star.

A star of magnitude 8.0 is easily measured, giving 0.028 volts per second, or say 10 scale divisions in about 7 seconds. The ordinary limit would be 9.0 mag., giving a current of 0.8×10^{-13} amp., and if this is to be measured to 1 per cent, the limiting current is about 10^{-15} amp. Stars still fainter could be observed, but under the usual conditions seventh magnitude with the 15-inch or ninth magnitude with the 40-inch represents an approximate limit.

17. *Determination of colors of stars.*—At present the photometer is being used principally for the measurement of color-indices of B stars which are also on the spectroscopic program of the 40-inch. The maximum color-sensitivity of a potassium-hydride cell is near λ 4600 A; but, by a suitable combination of blue and yellow filters, regions at about 4000 A and 5000 A may be made effective. This scheme has already been used by Bottlinger,[1] in his determination

[1] *Veröffentlichungen Berlin-Babelsberg*, **3**, Hft. 4, 1923.

of color-indices. At Yerkes we have by cut-and-try methods selected two pairs of yellow and blue filters; the first combination giving a color difference between Ao and Ko of 0.5 magnitude and the second about 1.5 magnitude, the total absorption for an A star being 1.0 and 3.0 mags., respectively. The intensities are about equalized for a B or an A star so that the measures are convenient for the objects under observation. Various suggestions have been made that two photo-electric cells of widely different color-sensitivities be used for such work, and, in fact, the photometer has been designed with that arrangement in view. Nevertheless, in spite of the loss of light involved in the use of filters and the small difference between the spectral regions utilized, a first-class single cell is much better than a combination of two fair ones. Up to the present I have not been able to secure a red-sensitive sodium or caesium cell which could be used to advantage for stellar photometry.

I am indebted to Professor Frost for the opportunity to undertake this project which was made possible by a grant from the Rosenwald Fund of the University of Chicago; also to different members of the staff for courtesies during my stay at Yerkes, particularly to Mr. C. T. Elvey who assisted in all the preliminary trials and who is now continuing the work in the manner exemplified in the following article of the *Journal*.

MADISON AND WILLIAMS BAY, WISCONSIN
July 14, 1931

PHOTO-ELECTRIC COLORS OF STARS OF EARLY TYPE

By C. T. ELVEY

ABSTRACT

The colors of 153 stars of early spectral classes have been determined with the photo-electric photometer attached to the 40-inch telescope of the Yerkes Observatory. The effective wave-lengths for the yellow and the blue magnitudes are λ 5100 and λ 3850. The scale of colors is 3.2 times that of K. F. Bottlinger. A comparison with Bottlinger's colors gives an average deviation of ±0.044 magnitudes on the Yerkes scale.

Correlations show that the *color excesses* (reddening of the star's light) *should be attributed not only to selective scattering* by material in interstellar space *but to the total absorption of hydrogen* in the star's atmosphere. This is an effect of absolute magnitude, since the total absorption of the hydrogen lines is proportional to the Stark effect, which is in turn proportional to the density of the stellar atmosphere.

It is well known that there are many cases among the early-type stars in which the colors are abnormal for their spectral class: some are too red for their class, and others are too blue. The red stars, those with a positive color excess, have been the subject of a controversy in regard to the explanation of the phenomenon. One group attributes the reddening to the scattering of light in interstellar space, while another places the cause within the atmosphere of the star itself. The following observations have been obtained in the hope that they may throw some light upon the question.

The instrument used to record the colors of the stars was a photo-electric photometer attached to the 40-inch telescope. It was designed by Professor Joel Stebbins and is described in the preceding article of this *Journal*.

A dense pair of filters was used, the transmissions of which were roughly determined by photographing the sky spectrum with and without the filter on a calibrated plate. The effective wave-length through Y_2 is about λ 5100, and the limits are, approximately, λ 4600 and λ 5900. The B_2 filter has an effective wave-length of λ 3850, and the limits are approximately λ 3650 and λ 4200. The short wave-length limit of B_2 is uncertain, as the transmission of the optical system of the 40-inch telescope was only estimated.

The observing was confined to nights on which it seemed very certain that the sky was of uniform transparency as well as could

be told by a visual inspection. Whenever it was found that the readings became variable, larger than a 10 per cent variation, the observing was discontinued, for with the photo-electric photometer it is possible to detect variations of light far fainter than can be seen with the eye. The transparency of the sky was estimated on a scale as defined by: 3, poor for photometry; $3\frac{1}{2}$, fair; 4, good; and $4\frac{1}{2}$, excellent. No observations were made unless the transparency was 3 or better. The majority of the observations were taken on nights of good quality, 4 or $4\frac{1}{2}$ on the scale used.

In order that the colors of stars determined on one night may be compared with those of another, a set of standard stars was chosen; and usually two or more of these stars were observed on each night, or rather that part of the night on which the photometer was assigned to the telescope. The list of standard stars is the same as that of Greaves, Davidson, and Martin.[1] A list of stars somewhat more suitable for the problem might have been chosen, but it seems desirable to have a group of standard stars which may be used to tie all photometric observations together. Some of the stars may have to be dropped from the list and others added as the observations may indicate. For instance, the Standard Star No. 14, 12 α Canum Venaticorum, is a variable star with a variable spectrum; and it is not at all certain that the color of such an object will be constant.

As soon as a standard star has been observed enough times to give an idea of its color or its brightness, it may be used as a measure of the transparency of the sky. This has proved quite satisfactory in the latter part of the work, and one may even observe a progressive change in the transparency of the sky on a given night when it is not noticeable to the eye.

At first the stars were observed in small groups, each in the following order, *a-b-c-d-d-c-b-a*, and then the various groups were to be connected so that all stars on the program could be compared with each other. The plan was abandoned in favor of that of employing the standard stars. A star was observed only once during the night, except for the standards, since one observation on each of two nights was better than two on the same night. Each observation of a star consisted of two determinations of the brightness

[1] *Monthly Notices of the Royal Astronomical Society*, **90**, 107, 1929.

through the yellow filter, four through the blue, four through the yellow, four blue, and then two yellow. These were divided into two sets for computing the color, and then the mean of the two sets taken. Each observation was reduced to outside the atmosphere and corrected for the transparency of the night by the following method.

In order to make use of a set of extinction tables, certain factors were introduced, since the tables were constructed for a different wave-length. The extinction as given in the tables is $E=0^{M}.2$ sec z, and introducing the factor we have $E_y=f_yE$, and $E_b=f_bE$. The color is then given by the equation,

$$C=(Y-E_y)-(B-E_b)-T \text{ sec } z \text{ ;}$$

or the equation may be put in the form:

$$C=(Y-B)-(f_y-f_b)E-T \text{ sec } z \text{ ,}$$

where Y, and B are the observed magnitudes of the star through the yellow and the blue filters; f_y, and f_b, the factors to be applied to the extinction referring to a sky transparency of $4\frac{1}{2}$; and T, the correction at the zenith for reducing the transparency of the sky to that of a standard night. The extinction factor (f_y-f_b) is found by observations of pairs of circumpolar stars. This was checked also by several observations of the same standard star on the same night. The resulting mean value of the factor (f_y-f_b) was adopted as 1.5 for the pair of filters designated Y_2B_2.

All of the observations of standard stars were reduced to outside the atmosphere without considering the variation of the transparency. When a sufficient number of observations had been obtained for the standard stars, a test was made to see the effect of the transparency of the night upon the colors. Early in the observing it had been noticed that the colors were very much affected by the transparency of the various nights, and thereafter the photometer was used only on the best nights. The color of each standard star was reduced empirically to the color of a given star and a diagram made by plotting the colors as ordinates and the estimated transparencies of the sky as the abscissae. The resulting curve is that given on the left-hand side of Figure 1. Since (T sec z) and the large variations

of the estimates of the transparency from night to night enter into the plot of the observations, the value of T was determined for each observation and the mean taken for a given night. The curve on the right-hand side of Figure 1 shows the variation of T with the estimated transparency. The agreement of the observations is very much better after the adjustment of the scale of transparency to a uniform system. This curve has been used to reduce the observed colors of a given night to those of a standard night, i.e., to a value

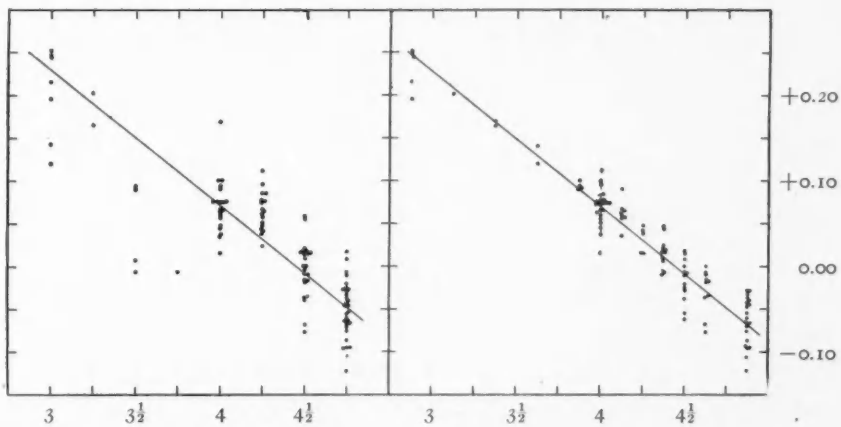


FIG. 1.—Plot showing the variation of color with the transparency of the sky. Ordinates are colors expressed in magnitudes and the abscissae are transparencies. The left-hand diagram is a plot using the eye estimates of the transparency of the sky made at the time of observing, and the right-hand diagram is one made after the scale of transparency has been adjusted by means of the standard stars.

of the transparency equal to $4\frac{1}{2}$. With this curve and the aid of the observed colors of the standard stars on later nights, it has been possible to make a measure of the transparency of the sky instead of merely estimating it, which is very difficult when one is comparing nights separated by several months, or when comparing moonlit nights with dark ones.

The colors of all stars determined are listed in Table I under the column headed "C." A comparison of my colors with those of K. F. Bottlinger[1] is made in order to show the order of accuracy that might

[1] *Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg*, Band 3, Heft 4, 1923.

be expected. As has been noticed by other investigators, it is quite possible that errors in the colors, originating in the atmosphere, may be present in all or part of the observations from any one place. The comparison is shown graphically by plotting Bottlinger's colors as ordinates and mine as abscissae in Figure 2. This diagram readily shows the difference in the two scales, the Yerkes colors being on a scale 3.2 times that of Bottlinger. This is due to the choice of the dense filters. The average deviation of the points from the mean curve, on the Yerkes scale, is ±0.044 magnitudes. This may, per-

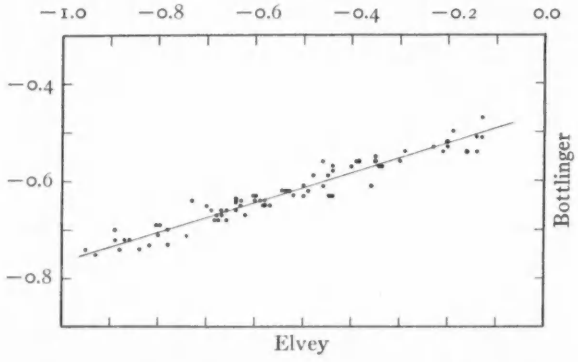


FIG. 2.—A comparison of the observed colors with those determined by Bottlinger for the same stars.

haps, seem a little large for determinations made with as precise an instrument as a photo-electric photometer; but it is not the instrument which limits the accuracy, but the transparency of the sky. This is very evident from a consideration of the large corrections that have to be applied to the colors to reduce the observations to a standard night. Of course, the accuracy of a comparison of the colors of two stars very near to each other is much greater.

In Figure 3 all of the stars in Table I have been plotted according to their positions in the sky, and the color excess is indicated by the size of the dot. A curve is drawn showing the position of the galactic equator. It is noticed that most of the stars with large positive color excesses are near the galactic plane. The principal exception is the star 41 θ' Orionis, which has next to the largest color excess and is

located in the Orion nebula. No doubt its color is much influenced by nebulous matter.

In order to compare the observed colors of the stars of the various spectral types, theoretical colors were computed for each spectral class. It was assumed that the stars radiate as black-bodies and that the temperature scale is that given by Russell, Dugan, and Stewart.[1] The stars of class O were all grouped together and assigned a temperature of 27,000°. The colors were computed for monochro-

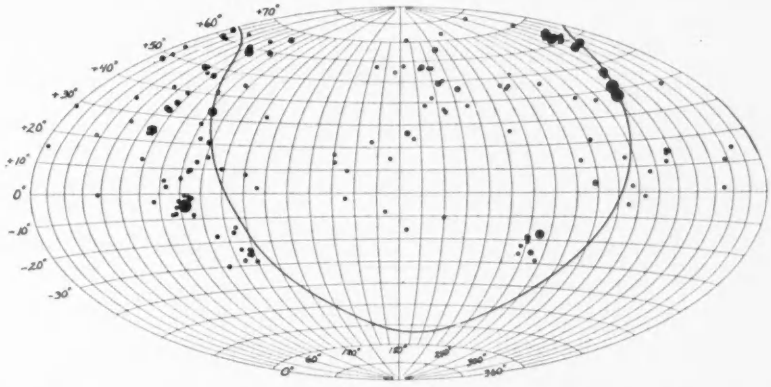


FIG. 3.—A chart of the stars showing the color excess by the size of the dot. The largest dots are stars that are too red by 0.75 mag. or larger. The curve is a trace of the galactic equator.

matic radiation at the effective wave-lengths for the pair of filters (λ 3850 and λ 5100) and are represented by the curve in Figure 4.

The observed colors of the stars in Table I have been plotted in Figure 4 according to their spectral types. It is seen at once that the spread of the colors for the B stars is very large, some of them having colors about the same as those of spectral types A and F. It is interesting to note that the bluest stars of each class fall very nearly on the smooth curve formed by the theoretical colors for the various temperatures. This would seem to indicate that at least a few of the stars radiate as black-bodies for the temperatures assigned to them, and that we may look to one of the several causes of the reddening of starlight for the deviations of the colors observed.

[1] *Astronomy*, II, 753.

TABLE I

Boss No.	Star	α	δ	m	Spec.	C	E	Remarks
10.....	21 α And	0h03m	+28°32′	2.2	Aop	−0.58	−0.03	Standard star No. 1
27.....	88 γ Peg	08	14 38	2.9	B2	.82	+ .08	Standard star No. 2
103.....	15 κ Cas	27	62 23	4.2	Bo	.50	.46	
122.....	17 ζ Cas	31	53 21	3.7	B3	.74	.13	
152.....	22 ο Cas	39	47 44	4.7	B2	.63	.27	
172.....	25 ν Cas	43	50 25	5.0	Bo	.58	.02	
199.....	27 γ Cas	□ 51	60 11	2.2	Bop	.66	.30	
314.....	37 δ Cas	1 19	59 43	2.8	A5	.23	.08	Standard star No. 3
419.....	45 ε Cas	47	63 11	3.4	B3	.68	.19	
428.....	6 β Ari	1 49	20 19	2.7	A5	.20	.11	Standard star No. 4
482.....	4 β Tri	2 04	+34 31	3.1	A5	.21	.10	Standard star No. 5
604.....	82 δ Cet	34	− 0 06	4.0	B2	.88	.02	
643.....	41 Ari	2 44	+26 51	3.7	B8	.59	.07	
708.....	26 β Per	3 02	40 34	(2.3)	B8	.50	.16	
742.....	29 Per	11	49 51	5.3	B3	.64	.23	
744.....	31 Per	12	49 43	5.1	B3	.60	.27	
781.....	2 H Cam	21	59 36	4.4	Bop	− .14	.46	
786.....	+58°607	22	58 32	4.8	Aop	+ .16	.71	
838.....	39 δ Per	36	47 28	3.1	B5	− .64	.15	Standard star No. 6
844.....	38 ο Per	38	31 58	3.9	B1	.58	.35	
851.....	16 Tau	39	23 59	5.4	B5	.58	.21	
852.....	17 Tau	39	23 48	3.8	B5p	.67	.12	
856.....	19 Tau	39	24 10	4.4	B5	.66	.13	
860.....	20 c Tau	40	24 04	4.0	B5	.63	.16	
865.....	23 d Tau	40	23 39	4.2	B5	.60	.19	
869.....	25 η Tau	42	23 48	3.0	B5p	.64	.15	
877.....	27 f Tau	43	23 45	3.8	B8	.63	.03	
879.....	28 Tau	43	23 51	5.2	B8p	.61	.05	
894.....	44 ζ Per	48	31 35	2.9	B1	.54	.39	
896.....	+62°628	49	62 47	4.9	B9	.48	.12	
910.....	45 ε Per	51	39 43	3.0	B1	.80	.13	
913.....	46 ξ Per	52	35 30	4.0	Oe5	.66	.33	
920.....	35 λ Tau	3 55	12 13	(3.8)	B3	.64	.23	
947.....	48 c Per	4 01	+47 27	4.0	B3p	.58	+ .29	
1079.....	48 ν Eri	31	− 3 33	4.1	B2	.94	− .04	
1139.....	9 Cam	44	+66 10	4.4	Bo	.60	+ .36	
1147.....	3 π4 Ori	46	5 26	3.8	B3	.83	.04	
1159.....	8 π5 Ori	49	2 17	3.9	B3	.87	.00	
1195.....	11 Cam	4 57	58 50	5.3	B3p	.60	.27	
1204.....	10 η Aur	5 00	+41 06	3.3	B3	.68	+ .19	
1231.....	69 λ Eri	04	− 8 53	4.3	B2	.95	− .05	
1250.....	19 β Ori	10	8 19	0.3	B8p	.69	− .03	
1262.....	20 τ Ori	13	6 57	3.7	B5	.64	+ .15	
1301.....	28 η Ori	19	− 2 29	3.4	B1	.89	.04	
1303.....	24 γ Ori	20	+ 6 16	1.7	B2	.89	.01	
1304.....	112 β Tau	20	28 31	1.8	B8	.60	.06	
1333.....	25 χ Aur	26	+32 07	4.9	B1	.23	.70	
1339.....	34 δ Ori	27	− □ 22	2.5	Bo	.88	.08	
1353.....	37 φ′ Ori	29	+ 9 25	4.5	Bo	.80	.16	
1357.....	39 λ′ Ori	30	+ 9 52	3.7	Oe5	− .86	0.13	
1363.....	41 θ′ Ori	30	− 5 27	5.4	Oe5	+ .23	1.22	
1366.....	44 ι Ori	31	5 59	2.9	Oe5	− .91	0.08	
1370.....	46 ε Ori	31	− 1 16	1.8	Bo	.93	.03	
1375.....	123 ζ Tau	32	+21 05	3.0	B3p	.78	.09	
1389.....	48 σ Ori	34	− 2 39	3.8	Bo	.92	.04	
1398.....	50 ζ′ Ori	36	2 00	2.0	Bo	.95	.01	
1435.....	53 κ Ori	43	− 9 42	2.2	Bo	.84	.12	
1438.....	134 Tau	44	+12 37	4.9	B9	.51	.09	
1478.....	34 β Aur	52	44 56	2.1	Aop	.30	.25	
1507.....	62 χ2 Ori	5 58	20 08	4.7	B2p	.44	.46	
1525.....	67 ν Ori	6 02	+14 47	4.4	B2	.80	.10	
1601.....	1 ζ CMa	16	−30 01	3.1	B3	.69	.18	
1609.....	2 β CMa	18	−17 54	2.0	B1	.82	.11	
1706.....	15 S Mon	36	+ 9 59	4.7	Oe5	.78	.21	
1793.....	20 ι CMa	52	−16 55	4.4	B5	.59	.20	
1804.....	21 ε CMa	55	28 50	1.6	B1	.77	.16	
1817.....	24 ο2 CMa	59	23 41	3.1	B5p	.66	.13	
1819.....	23 γ CMa	6 59	15 29	4.1	B5	.60	.19	
1872.....	27 CMa	7 10	26 10	4.7	B5p	.76	.03	
1877.....	28 ω CMa	11	26 35	3.8	B3	.72	.15	
1899.....	29 CMa	14	24 23	4.9	Oe	.81	.18	
1901.....	30 τ CMa	14	24 47	4.4	Oe5	.81	.18	
1934.....	31 η CMa	7 20	−29 06	2.4	B5p	− .63	+0.16	

TABLE I—*Continued*

Boss No.	Star	α	δ	m	Spec.	C	E	Remarks
1944	3 β CMi	7h22m	+ 8° 29′	3.1	B8	−0.54	+0.12	
1979	α Gem	28	32 06	2.0 2.8	Ao	.30	.25	Standard star No. 10
2071	13 ζ CMi	7 46	2 01	5.1	B8	.64	.02	
2694	30 η Leo	10 02	17 15	3.6	Aop	.53	.02	
2698	32 α Leo	03	+12 27	1.3	B8	.58	.08	Standard star No. 11
2788	29 δ Sex	24	− 2 14	5.2	B9	.43	.17	
2804	47 ρ Leo	28	+ 9 49	3.8	Bop	.75	.21	
2930	48 β UMa	10 56	56 55	2.4	Ao	.40	.15	Standard star No. 12
2972	68 δ Leo	11 09	+21 04	2.6	A3	.19	.21	Standard star No. 13
3055	21 θ Cra	32	− 9 15	4.8	B9	.49	.11	
3101	94 β Leo	44	+15 08	2.2	A2	.20	.24	
3117	64 γ UMa	11 49	54 15	2.5	Ao	.39	.16	
3190	69 δ UMa	12 10	+57 35	3.4	A2	.24	.20	
3191	4 γ Crv	11	−16 59	2.8	B8	− .54	.12	
3224	12 Com	17	+26 24	4.8	F5	+ .26	.26	
3281	5 κ Dra	29	70 20	3.9	B5p	− .64	.15	
3283	23 Com	30	23 11	4.8	Ao	.33	.22	
3363	77 ε UMa	50	56 30	1.7	Aop	.40	.15	
3370 3371	12 α CVn	51	38 52	2.9 5.4	Aop	− .48	.07	Standard star No. 14
3382	78 UMa	12 56	56 55	4.9	Fo	+ .17	.33	
3447	20 CVn	13 13	41 06	4.7	Fo	+ .04	.20	
3474 3475	79 ζ UMa	20	+55 27	2.4 4.0	A2p A2	− .26	.18	
3476	67 α Vir	20	−10 38	1.2	B2	.76	.14	
3480	80 UMa	21	+55 30	4.0	A5	− .08	.23	
3511	H.D. 118216	30	37 42	5.0	Fo	+ .06	.22	
3512	24 C Vn	30	49 32	4.6	A3	− .06	.34	
3518	25 C Vn	33	36 48	4.9	Fo	+ .04	.20	
3566	85 η U Ma	13 44	49 49	1.9	B3	− .67	.20	Standard star No. 15
3626	11 α Dra	14 02	64 51	3.6	Aop	.26	.29	
3666	19 λ Boo	13	46 33	4.3	Ao	.21	.34	
3722	27 γ Boo	14 28	38 45	3.0	Fo	.09	.07	
3928	13 γ U Mi	15 21	72 11	3.1	A2	.44	.00	Standard star No. 16
3961	5 α Cr B	30	+27 03	2.3	Ao	.34	.21	Standard star No. 17
4062	6 π Sco	53	−25 50	3.0	B2	.74	.16	
4066	7 δ Sco	15 54	22 20	2.5	Bo	.72	.24	
4086	8 β′ Sco	16 00	−19 32	2.9	B1	.66	.27	
4089	6 υ Her	00	+46 19	4.6	B9	.52	.08	
4093	9 ω′ Sco	01	−20 24	4.1	B2	.71	.19	
4112	11 φ Her	06	+45 12	4.2	B9p	.49	.11	
4158	20 σ Sco	15	−25 21	3.1	B1	.59	.34	
4162	22 τ Her	17	+46 33	3.9	B5	.73	.06	
4183	7 χ Oph	21	−18 14	4.8	B3p	.24	.63	
4218	23 τ Sco	16 30	−28 01	2.9	Bo	.98	− .02	
4459	55 α Oph	17 30	+12 38	2.1	A5	.14	+ .17	Standard star No. 18
4479	85 ι Her	37	46 04	3.8	B3	.68	.19	
4548	67 Oph	17 56	2 56	3.9	B5p	.51	.28	
4590	102 Her	18 04	20 48	4.3	B3	.68	.19	
4722	3 α Lyr	18 34	38 41	0.1	Ao	.34	.21	Standard star No. 19
4858	17 ζ Aql	19 01	+13 43	3.0	Ao	.35	+ .20	Standard star No. 20
4859	16 λ Aql	01	− 5 02	3.6	B9	.64	− .04	
4887	21 Aql	09	+ 2 07	5.1	B8	.57	+ .09	
4897	20 η Lyr	10	+38 58	4.5	B3	.64	.23	
5004	41 ι Aql	32	− 1 31	4.3	B5	.58	.21	
5062	53 α Aql	19 46	+ 8 36	0.9	A5	.13	.11	Standard star No. 21
5170	28 b_2 Cyg	20 05	36 33	4.8	B2p	.65	.25	
5199	1 κ Cep	12	77 25	4.4	B9	.51	.09	
5208	34 P Cyg	14	37 43	4.9	B1p	− .10	0.83	
	H.D. 194279	20	40 26	7.0	Bo	+ .48	1.44	N.G.C. 6910
5272	2 ε Del	28	10 58	4.0	B5	− .60	0.19	
5291	β Del	33	14 15	3.7	F5	+ .22	.22	
5310	9 α Del	35	15 34	3.9	B8	− .48	+ .18	
5320	50 α Cyg	38	44 55	1.3	A2p	.45	− .01	Standard star No. 22
5323	11 δ Del	39	14 43	4.5	A5	.08	+ .23	
5361	55 Cyg	20 46	45 45	4.9	B2	.16	.74	
5480	5 α Cep	21 16	62 10	2.6	A5	.13	.11	Standard star No. 23
5512	H.D. 204172	22	36 14	5.8	Bo	.64	.32	
5532	8 β Cep	27	70 07	3.3	B1	.87	.06	
5563	9 Cep	35	61 38	4.9	B2p	.36	.54	
5565	+55°2617	36	57 02	5.6	Oe5	− .48	.51	
5608	μ Cep	21 43	+60 40	4.5	A2p	+0.05	+0.49	

TABLE I—*Continued*

Boss No.	Star	α	δ	m	Spec.	C	E	Remarks
.....	H.D. 208185	21^h50^m	+62°38′	7.7	B3	−0.36	+0.51	
.....	H.D. 208218	50	62 13	6.8	B1	.26	.67	N.G.C. 7160
.....	H.D. 208392	51	62 08	7.1	B3	.37	.50	
.....	H.D. 208761	21 53	62 25	8.7	B5	.48	.31	
5687.....	19 Cep	22 02	61 48	5.2	Oe5	.62	.37	
5719.....	22 λ Cep	08	58 56	5.2	Od	.44	.55	
5777.....	52 π Aqr	20	0 52	4.6	B1p	.74	.19	
5844.....	10 Lac	35	38 32	4.9	Oe5	.84	.15	
5853.....	42 ζ Peg	22 36	10 19	3.6	B8	.57	.09	
5944.....	54 α Peg	23 00	+14 40	2.6	A0	−0.46	+0.09	Standard star No. 24

We may divide the causes of the reddening of starlight into two classes: those causes which are intimately connected with the star itself, and those which are independent of the star. In the first class are: (1) stars do not radiate like black-bodies; (2) absorption by line and bands in the stellar atmosphere; (3) scattering by an extended atmosphere. In the second class is scattering of light, after it has left the star, by material such as nebulosity and interstellar matter.

The question whether the stars radiate as black-bodies has never been completely answered, for all observations on that problem have to be corrected for scattering and absorption of the starlight after it leaves the photosphere. However, this may not be of a serious concern, for we can compare stars of the same spectral types, and one would not expect such very large deviations in the character of the continuous background within the group of stars. In the case of the giants and the dwarfs of late types, there is a large difference in the mean densities of the objects and yet comparatively small changes in the temperature. If we can assume that the observed colors of the early-type stars indicate temperatures, we see that there are early B stars that have a temperature of A- and F-type stars. Miss C. H. Payne[1] has shown that the low temperatures are inadequate to produce the corresponding spectra by thermal ionization. It would seem that we must look to other causes for at least a part of the reddening of starlight.

The influence of absorption lines and bands originating in the atmosphere of the star has been discussed by many, especially in the case of the late-type stars having the absorption bands of TiO_2.[2]

[1] *Stars of High Luminosity*, p. 114.

[2] Bottlinger, *Handbuch der Astrophysik*, **2**, erste halfte, 361, 1929.

The influence of the hydrogen lines on the colors of stars has perhaps not been fully appreciated. It has been shown[1] that the wings of the hydrogen lines are much more extensive than has been thought,

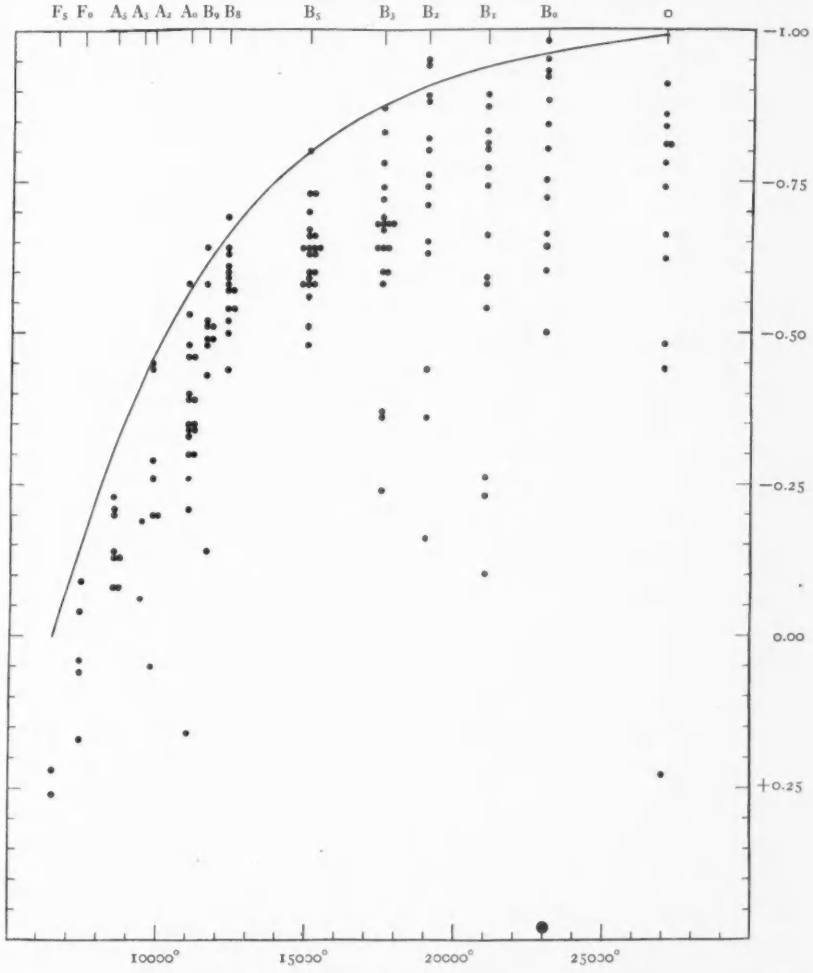


FIG. 4.—A plot of the colors of the stars according to their temperatures. The curve represents the theoretical colors for the given temperatures.

the overlapping occurring as early in the series as $H\gamma$ and $H\delta$. The hydrogen lines would have very little effect upon the magnitude of

[1] C. T. Elvey and O. Struve, *Astrophysical Journal*, **72**, 277, 1930.

a star through the yellow filter, λ 4600–λ 5900, but would influence quite appreciably those through the blue filter, λ 3650–λ 4200. Since the effective wave-length of the B_2 filter is at λ 3850, which is between the lies $H\eta$ and $H\zeta$, there would be few stars not influenced by these lines. The limits of the filter include the lines from $H\delta$ to the end of the series. In this connection it is interesting to note that the observed colors for some of the bright narrow-lined stars are among the bluest in their spectral classes, fitting closely to the theoretical curve. The supergiant star, 50 α Cygni, has a color excess of −0.01, and several stars of the same type have excesses of +0.25 mag. Also, 19 β Orionis of spectral class B8, with very narrow lines, has a very small color excess, in comparison with others of the class.

An extensive atmosphere, such as might be caused by very rapid rotation of the star, would tend to produce scattering of the light coming from the photosphere. This problem would have to be studied statistically with a large number of stars in order to minimize the other effects. One may look for this effect also in those stars of class B having emission lines. O. Struve[1] has shown that rapid rotation is the explanation of the emission lines. Hence, a star with narrow single emission lines is one that is rotating rapidly about an axis in the line of sight, and we would expect the gaseous envelope to have very little effect upon the light coming from the star. On the other hand, the stars with wide emission lines are probably rotating about an axis perpendicular to the line of sight, and we see the star through the nebulous shell. We might expect, then, to find the stars with broad emission lines to be redder than those of the same spectral class with narrow lines.

The selective scattering of light in interstellar space has been investigated by R. J. Trumpler,[2] who finds from his studies of the open clusters that there is a selective scattering of 0.32 mag. per 1000 parsecs on the Harvard scale of color indices. The absorbing material is thought to form a rather thin sheet of matter extending along the galactic plane. P. van der Kamp[3] obtains a thickness of 175 parsecs for the absorbing layer.

[1] *Astrophysical Journal*, **73**, 94, 1931.

[2] *Bulletins of the Lick Observatory*, **14**, 154, 1930; *Astronomical Society of the Pacific*, **42**, 267, 1930.

[3] *Astronomical Journal*, **40**, 145, 1930; **41**, 84, 1931.

At first it seemed that a study with a photo-electric photometer of the colors of the bright stars of early spectral classes would provide suitable material for determining the amount of selective scattering, but one finds that the other factors become increasingly more important for these stars. Furthermore, the distances of the individual stars of early spectral classes are not known with any accuracy. They are too far for trigonometric determinations of their parallaxes, and the spectroscopic methods are statistical and subject to considerable doubt. The criterion of the spectroscopic method is the width of the spectral lines, sharp or diffuse, and this un-

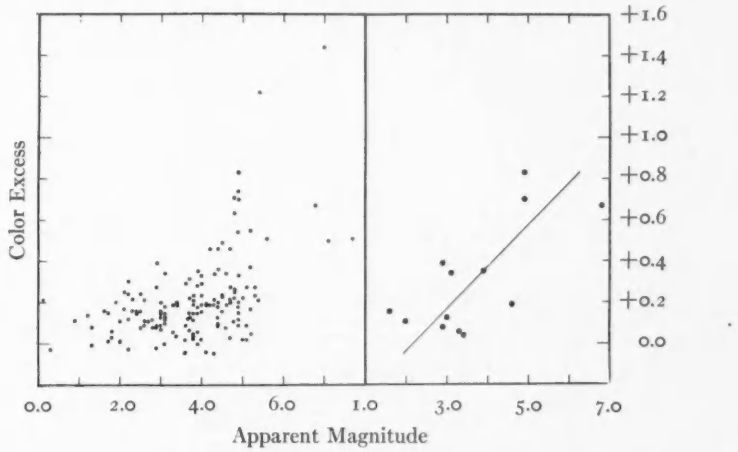


Fig. 5.—A diagram of the color excess of a star plotted against its apparent magnitude.

doubtedly reflects the amount of rotation of the star, as O. Struve[1] has shown. Another difficulty is that the absorption is a statistical matter; and when dealing with the closer objects, the local irregularities in the distribution of the layer become deciding factors.

In view of the foregoing, it was decided that a correlation of the color excesses with the apparent magnitudes would be as indicative of scattering by interstellar matter as any other method. Figure 5 shows a plot of the color excesses with the apparent magnitudes of the stars. In the left-hand side of the figure is a plot of all of the stars observed, a few stars of spectral class as late as F5 are included,

[1] *Observatory*, **54**, 80, 1931.

and hence there is a large range in the absolute magnitudes of the stars. The dispersion in the colors increases with the faintness of the star, there being no stars brighter than the fourth magnitude with large color excesses while there are several among those fainter. This is what would be expected for a scattering of light in interstellar space for a group of relatively bright stars with a considerable range in their absolute magnitudes. Within any one spectral

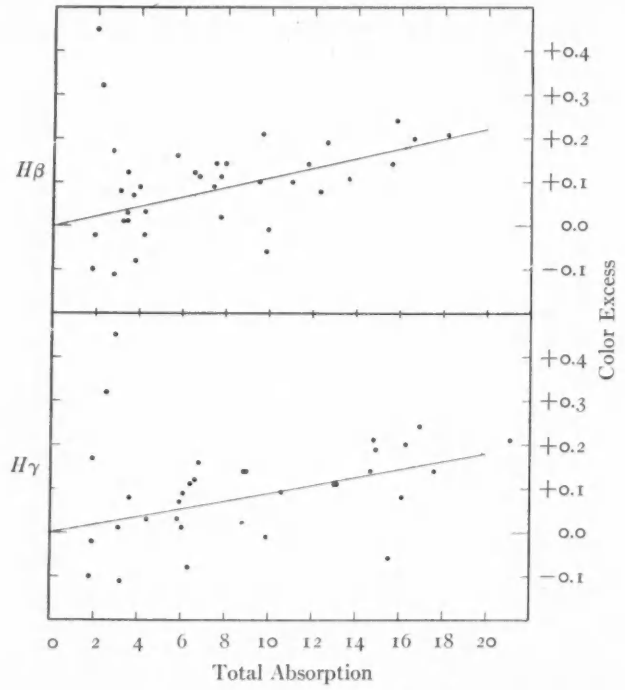


FIG. 6.—A correlation of the color excess of a star with the total absorption of a hydrogen line in its spectrum.

type the scatter in absolute magnitude is much smaller, and consequently a much better correlation would be expected. There are, perhaps, an insufficient number of stars in any class for a statistical treatment, but an example is shown by plotting the color excesses against the apparent magnitudes for spectral type B2, right-hand side of Figure 5. This relationship shows a scattering which is within that of the absolute magnitudes of the spectral type. A color ex-

cess of about 0.2 mag. per magnitude is indicated for this group of stars.

In this connection it is interesting to note that the star H.D. 194279, which is in the cluster N.G.C. 6910 and has a spectrum of type Bo, has a color excess of +1.44. R. J. Trumpler[1] gives the distance of this star as 2100 parsecs. The stars H.D. 208185, 208218, 208392, and 208761 are in the region of the cluster N.G.C. 7160 and are probably in it. They give the color excesses +0.51, +0.67, +0.50, and +0.31 mag., respectively. The distance to this cluster is given by Trumpler[2] as 1030 parsecs.

In order to test the effect of the hydrogen lines on the colors of the stars, the colors were corrected for distance as given by the parallaxes of D. L. Edwards,[3] assuming a selective absorption the same as that given by Trumpler, and adjusting it to the scale of colors determined in this paper, i.e., 0.48 mag. per 1000 parsecs. The color excesses corrected for distance are plotted against the total absorptions of $H\beta$ and $H\gamma$ in Figure 6. With the exception of two stars of spectral class B, there seems to be a fair correlation of the color excesses with the total absorption of the hydrogen lines. As mentioned above, the supergiant stars α Cygni and β Orionis fall very close to the computed colors for their spectral classes, and the stars of the same classes with normal hydrogen lines show a decided color excess.[4]

In conclusion, we may say that there are at least two factors tending to produce a reddening of the light of a star of early type, namely, the scattering of the light by matter in interstellar space, and the absorption by the hydrogen lines in the violet end of the spectrum.

I am very much indebted to Professor Joel Stebbins for his discussions and criticisms of this paper.

YERKES OBSERVATORY
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[1] *Astronomical Society of the Pacific*, **42**, 267, 1930.

[2] *Lick Observatory Bulletin*, **14**, 154, 1930.

[3] *Monthly Notices of Royal Astronomical Society*, **87**, 364, 1927.

[4] See, also, a paper by Miss C. H. Payne, *Proceedings of the National Academy of Sciences*, **14**, 296, 1928.

NOTE ON CHANGES IN THE LUMINOSITY FUNCTION WITH DISTANCE FROM THE SUN[1]

By FREDERICK H. SEARES

ABSTRACT

Variations in the luminosity function.—The *rapidly changing space density* of stars found for points *near the sun* (*Contribution* No. 436, Table VI) *denotes* an error either in the star counts used for the calculation or in the assumed constancy of the luminosity function. Error in the counts would imply an inadmissible error in the magnitude scale; the alternative to be accepted is a *decrease in the frequencies of high-luminosity stars with increasing distance* from the sun. Evidence from *van Rhijn's discussion* of the luminosity function *confirms* the dependence on distance and *suggests a change with galactic latitude.* These *conclusions* are *in harmony with the structure of the local system.* The thin stratum of *helium stars* concentrated in this system, and possibly part of the A stars, *accounts* qualitatively at least for the *excess of high luminosities* near the sun. Elsewhere the mean density of these stars is low; hence, the frequencies for $M<0$ decrease with increasing distance. The *exaggerated distance effect* occurring in *high galactic latitudes* because of the thinness of the stratum of helium stars *introduces a dependence* of frequency *on latitude* for stars situated at the same distance from the sun. *Allowance for the variation* in the frequencies *will flatten* the *curve of space density* found with a constant luminosity function, decreasing densities within 200 parsecs of the sun and increasing those at greater distances.

The space density of stars near the sun found by the numerical solution for the density function given in *Contribution* No. 436[2] differs widely from the results of Kapteyn and van Rhijn[3] and from those of a later investigation by van Rhijn.[4] The number of stars per cubic parsec drops from 0.166 near the sun to 0.044 at 125 parsecs, while Kapteyn and van Rhijn find a nearly constant density of 0.045 out to about 100 parsecs. A 4-to-1 change in so short a distance can be rejected on the grounds of probability alone. On the other hand, a constant, or, at most, a slowly varying, density near the sun has the support of sound observational data and must be substantially correct; it depends on the numbers of stars in successive intervals of magnitude and proper motion, the mean-parallax formula,

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 437.

[2] *Astrophysical Journal*, **74**, 268, 1931.

[3] *Mt. Wilson Contr.*, No. 229, Table Ib; *Astrophysical Journal*, **55**, 246, 1922.

[4] *Groningen Publication*, No. 38, p. 15, 1926. See also Seares, *Mt. Wilson Contr.*, No. 273, Table VI; *Astrophysical Journal*, **59**, 337, 1924. The densities near the sun found in *Contribution* No. 273 show some change with distance, but nothing like that in *Contribution* No. 436, which assumes the luminosity function to be constant.

and the dispersion in parallax; errors in the magnitude scale are eliminated, and uncertainties in the proper-motion data do not seriously affect results for the region in question.

The direct observational data for the solution in *Contribution* No. 436 are counts of stars for successive intervals of magnitude. The only other datum is the luminosity function of Kapteyn and van Rhijn[1], which, in the mean, for the luminosities involved, is open to little question. For lack of definite information, however, it was necessary to assume that this function is the same at all distances. Combined with the calculated densities, the constant function gives a practically perfect representation of the star counts (*Contribution* No. 436, third column of Table II). Since the relative densities are certainly in error, the source of the discordance must lie either[2] in the counts themselves or in the assumed constancy of the luminosity function ϕ. Accept first the constancy of ϕ and consider the star counts.

Constant density for a hundred parsecs implies unit values for the relative densities d_0, d_2, . . . , in successive shells (thickness = 0.2 in log ρ) over the same range in distance. Equation (9), supplemented by Table VII and Figure 1, all of *Contribution* No. 436, shows that δ_m would then be sensibly unity for all the brighter stars,[3] and that log δ_m could not differ from zero by more than 0.02 or 0.03, even for $m = 5.5$. To this limit, therefore, the star ratio should have practically the theoretical value of 3.98 (logarithm = 0.6) corres-

[1] *Loc. cit.* For stars of low luminosity the frequencies given by the function of Kapteyn and van Rhijn are much too low. The relative densities found with this function are little affected, but the absolute values expressed in stars per cubic parsec, including those given in the text, are seriously in error. See Seares, *Mt. Wilson Contr.*, No. 273, p. 26; *Astrophysical Journal*, **59**, 335, 1924; and van Rhijn, *Groningen Publication*, No. 38, p. 16, 1926.

[2] A third possibility would throw the discordance on non-selective absorption produced by an obscuring cloud localized about the sun. Constant density near the sun could thus be obtained, but its value would be four times that found from mean parallaxes and proper-motion data. The magnitude term in the parallax formula would of course refer to magnitudes as affected by absorption, but the formula would be applied to other data involving magnitudes similarly affected; the absorption effect, like scale error, would thus be eliminated.

[3] The symbol δ_m indicates the weighted mean value of d for stars of magnitude m and is equal to $R/3.98$, where R is the star ratio for stars of magnitude $m+1$ and magnitude m.

ponding to constant density. Actually the values of log δ_m for the counts used range from -0.06 to -0.08 for $m<6$. To account for this discrepancy by an error in the star counts, we must suppose that the magnitude scale requires a 10 per cent correction, having the general form $-0.10\,(m-5.5)$, where $m \lesseqgtr 5.5$.

There is not, however, the slightest observational justification for any such correction. The source of the counts may be traced through *Groningen Publications* Nos. 27 and 18 back to *Harvard Annals* **14** and **34**, which are the principal basis for the magnitudes of bright stars in the *Revised Harvard Photometry* (*H.A.* **50**). Comparisons of the various Harvard catalogues[1] with the extra-focal photographic measures of King and with the visual measures of the *Potsdam Durchmusterung* reveal the influence of color equation but no scale difference of the kind required. This result refers to the bright stars as a whole, while the counts used are for galactic latitude 0°. It is clear, however, that the grouping of the stars according to latitude has not artificially introduced an error of scale into the data for latitude 0°, for the counts over the whole sky (*Groningen Publication* No. 27, Table V) show the same peculiarity as those for the galactic plane: the star ratios are all definitely less than the value corresponding to constant density. The possibility of scale error therefore does not enter; while its alternative, a dependence of luminosity on distance, apparently must be admitted.

The nature of the dependence appears at once from Table VII of *Contribution* No. 436. The quantities tabulated have the form $DV\phi$ (density times volume times frequency) and were computed with the calculated densities, which for points close to the sun are too large. When the correct densities are substituted, the frequencies of the corresponding absolute magnitudes must be increased, in order that the star counts may still be represented. The stars affected are those of high luminosity; hence, highly luminous stars must be relatively more numerous near the sun than at greater distances. This conclusion has been stated elsewhere,[2] but the densities found in *Contribution* No. 436 give it emphasis.

[1] *Mt. Wilson Contr.*, No. 288, Tables IV–VI; *Astrophysical Journal*, **61**, 114, 1925; Zinner, *Veröffentlichungen der Remeis-Sternwarte zu Bamberg*, **2**, 65–72, 1926.

[2] *Mt. Wilson Contr.*, No. 347, p. 42; *Astrophysical Journal*, **67**, 164, 1928.

Since the results of Kapteyn and van Rhijn[1] were obtained by a method which gives not only the density but also the luminosity function itself at different distances from the sun, their data should show traces of the change in frequency with distance. The dependence of ϕ on distance does in fact appear, and is commented on by van Rhijn in his discussion of the luminosity function in *Groningen Publication* No. 38.[2]

The derivation of this function involves the preparation of a table similar to Table VII, *Contribution* No. 436. The total numbers of stars in successive intervals of magnitude and proper motion are assumed to be known. The mean parallax formula then gives the mean parallax of each group of stars having a specified magnitude and motion; and, with the aid of the dispersion in parallax, the members of the various groups can be distributed among the successive shells to which they belong. The summation of the frequencies for each μ, m being constant, gives finally the distribution for all the stars of magnitude m. The totals represent one column of the required table and should agree with the corresponding totals of Table VII. The two tables will not, however, be coextensive, because proper-motion data are not now available for stars at great distances.

Each line of the table thus prepared represents a section of the luminosity-curve,[3] the individual frequencies being multiplied by the volume and the density of a certain spherical shell. With the aid of the known volumes of the shells and the relative densities (d) found from the table itself, the different sections of the curve may be brought into coincidence (provided the luminosity function does not change with distance) and combined into a mean curve. Summation of the values of $\log d$ gives $\log D$, and the solution for lu-

[1] *Loc. cit.*

[2] My calculations for the luminosity function which are summarized in Table V and Figure 2 of *Contribution* No. 273 also show traces of the dependence on distance discussed in the present article. I have preferred, however, to refer chiefly to van Rhijn's monograph, because it gives details and is based on data that are later and much more extensive than those used for my calculations.

[3] In Table VII the abscissae of the different sections, i.e., the values of M corresponding to $\log \rho$ and the different values of m, shift one magnitude to the right in passing from any line to the next.

minosity and density is then complete, at least within the range of the data used. The constant density near the sun found by Kapteyn and van Rhijn was obtained in this way.

A dependence of ϕ upon distance will be signified by a lack of parallelism in the different sections of the luminosity-curve. A peculiarity of this sort is clearly shown in van Rhijn's discussion.[1] For stars of apparent magnitudes 5–12 the frequencies are, however, open to question because some of the mean parallaxes depending on small proper motions are extrapolations, and van Rhijn has preferred to interpret the lack of parallelism as evidence of systematic error in the parallaxes. A correction to these parallaxes removes much of the divergence between the curves. Nevertheless, the phenomenon still appears in the frequencies of the highly luminous stars, as may be seen from Table I.

This table was formed from a portion of van Rhijn's Table 12, which includes his systematic correction. Each line represents a section of the luminosity-curve, with the values of M aligned vertically. The lines of constant m run diagonally upward to the right, with the top diagonal in each section corresponding to $m = 2$. Further, the frequencies in Table I have been reduced to unit density with the aid of van Rhijn, Table 13. The tabulated quantities, which are logarithms of numbers of stars per cubic parsec of standard density, are therefore strictly comparable. Hence, a progressive change in the numbers of any column indicates a dependence of frequency on distance. For the lower luminosities, van Rhijn's systematic correction to the parallaxes has brought the frequencies in the different shells into agreement; but for $M < 0$ (< -5, Kapteyn's scale), the numbers in all latitudes decrease with increasing distance from the sun. Further, it is also to be noted that stars of the same luminosity, situated at the same distance but seen in different directions, are more frequent in low latitudes than in high.

Similar changes in the frequencies also appear in van Rhijn's Tables 20 and 21, derived by another method. Since trigonometric parallaxes alone were used, the range in distance is less and the variation with distance less clearly established. The mean parallax formula does not enter. Proper motions, however, were used in finding

[1] *Groningen Publication*, No. 38, p. 11.

the total numbers of stars in different intervals of parallax; otherwise, the clearly marked change in the frequencies with latitude could scarcely be questioned on any ground.

TABLE I

LUMINOSITY FUNCTION

LOGARITHMS OF THE NUMBERS OF STARS PER CUBIC PARSEC*

MED. LOG ρ	VISUAL M (KAPTEYN'S SCALE)						
	−8.44	−7.45	−6.46	−5.48	−4.50	−3.50	−2.52
	Latitude $0° \pm 20°$						
1.1	.....	.....	.....	.....	.....	6.41	6.85
1.3	.....	.....	.....	.....	6.44	6.60	6.93
1.5	.....	.....	.....	5.82	6.22	6.56	6.89
1.7	.....	.....	5.22	5.74	6.10	6.53	6.90
1.9	.....	4.83	5.21	5.60	6.11	6.53	6.89
2.1	3.93	4.53	5.04	5.62	6.09	6.54	6.91
2.3	3.64	4.32	5.02	5.56	6.09	6.56	6.93
2.5	3.30	4.25	4.88	5.52	6.10	6.58	6.98
2.7	.....	.....	4.76	5.50	6.13	6.65	7.05
2.9	.....	.....	.....	.....	6.28	6.76	7.20
	Latitude $\pm 20° \pm 40°$						
1.1	.....	.....	.....	.....	.....	6.43	6.78
1.3	.....	.....	.....	.....	6.33	6.54	6.90
1.5	.....	.....	.....	5.57	6.16	6.53	6.86
1.7	.....	.....	.....	5.65	6.05	6.51	6.90
1.9	.....	4.50	5.00	5.49	6.05	6.52	6.86
2.1	.....	4.23	4.84	5.47	6.05	6.51	6.85
2.3	3.31	4.07	4.74	5.45	6.04	6.50	6.88
2.5	3.08	3.78	4.67	5.40	6.03	6.52	6.89
2.7	.....	.....	.....	5.38	6.00	6.54	6.92
2.9	.....	.....	.....	.....	6.02	6.58	6.94
	Latitude $\pm 40° \pm 90°$						
1.3	.....	.....	.....	.....	6.38	6.43	6.90
1.5	.....	.....	.....	5.77	5.96	6.49	6.82
1.7	.....	.....	.....	5.36	6.00	6.45	6.87
1.9	.....	.....	4.68	5.40	5.99	6.48	6.81
2.1	.....	3.86	4.67	5.41	5.98	6.45	6.83
2.3	.....	3.78	4.69	5.35	5.80	6.43	6.82
2.5	.....	3.72	4.52	5.37	5.93	6.45	6.85
2.7	.....	.....	4.51	5.39	5.99	6.45	6.81
2.9	.....	.....	.....	5.36	5.97	6.48	6.81

* The characteristic −10 is to be appended to each tabular logarithm.

The results from *Contribution* No. 436 and these details from van Rhijn's calculations together afford excellent evidence of an excess of high-luminosity stars within a few hundred parsecs of the sun. Further, the distribution of luminosity seems also to depend on galactic latitude, although van Rhijn attributes this variation also to systematic errors in the data;[1] the variation, however, is in close agreement with other results, as will appear in a moment. How much of the curve is affected by distance cannot be stated at present. According to Table I, the giant stars alone are involved; but without van Rhijn's rather arbitrary correction, objects as faint intrinsically as the sun would also have shown changes in frequency.

These results are in complete harmony with our general knowledge of stellar distribution. The sun is not far from the center of an important local aggregation, which includes a large group of helium stars, arranged in a thin stratum, inclined a few degrees to the galactic plane and extending outward in all directions with gradually decreasing density for a thousand parsecs or so. The concentration of the highly luminous helium stars within the local system marks a tendency displayed by these objects everywhere in the sky; we find them, in general, only in groups, in the open clusters (and not in every cluster at that) or in the galactic star clouds. Near the sun their space density is comparable with that of stars of other spectral types; but for the heavens as a whole their mean density is excessively low. Since the distribution of luminosity is necessarily calculated from stars at no great distance from the sun, the computed frequencies for the most luminous stars are therefore relatively high. With increasing distance in or near the galactic plane the helium stars thin out, and we find a corresponding decline in the numbers of intrinsically bright stars. Toward the poles of the Milky Way the thinning-out is much more rapid; the change in frequency with distance sets in sooner, and a dependence of the luminosity function on galactic latitude thus makes its appearance.

For simplicity of statement the observed changes in the frequencies with distance and with latitude are attributed to the presence of helium stars in the local system, but other spectral types also may have some influence. In fact, part of the A stars, at least, display

[1] *Op. cit.*, p. 76.

some of the characteristics of distribution shown by the helium stars, and it is therefore probable that these stars also are a contributing factor.

Whether the observed changes are wholly a consequence of the structure of the local system, we have no means of knowing. If they are, we should expect most of the dependence on latitude to disappear at a thousand or fifteen hundred parsecs. In fact, Table I suggests that beyond some such limit the distribution of luminosity is indeed the same in all latitudes, and possibly the same at all distances, except as it may be modified by other localized groups of high-luminosity stars; but in view of van Rhijn's reservations in respect to the data on proper motion and mean parallax, the question remains speculative for the present.

As a practical question affecting the star counts, the variations in the luminosity function seem of little consequence, since only a few hundred stars appear to be involved; but from the standpoint of influence on the space density of stars, the matter is important. We cannot use the distribution of luminosity found from stars within seven or eight hundred parsecs of the sun, where the mean frequencies of certain absolute magnitudes are high, to calculate the density in other regions where the corresponding frequencies are low and obtain correct results. Even near the sun the errors are large (*Contribution* No. 436, Table VI), because the frequencies are rapidly changing. At great distances the densities are also considerably in error, because in remote regions stars must be of high luminosity in order to be seen, and for distances beyond 200 parsecs the frequencies hitherto used for such stars are too high. The corresponding densities are therefore too low, and the true decrease in density, outward from the sun, must be less rapid than that found with the constant luminosity function ordinarily used.

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MOUNT WILSON OBSERVATORY
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MEAN PARALLAXES AND THE LUMINOSITY FUNCTION[1]

By FREDERICK H. SEARES

ABSTRACT

The mean-parallax formula.—Data on the proper motions and the parallaxes of faint stars recently collected by van Maanen and Willis are well represented by

$$\log \bar{\pi} - \log \mu = a + bH, \qquad H = m + 5 \log \mu,$$

a type of mean-parallax formula established in *Contribution* No. 273, which has different values of b for $H \lesseqgtr 7.5$. The results confirm the form of the equation and provide improved values of the coefficients for the case $H > 7.5$. The adopted formula is equation (12), the zero-point (provisional) for which depends on the measured parallaxes of 76 stars used in *Contribution* No. 273.

Dependence of linear velocity on absolute magnitude.—The mean parallaxes given by van Maanen and Willis were derived from τ components of the proper motions and an assumed linear velocity. The deviations of the parallaxes from the formula are correlated with the absolute magnitudes. The correlation affords new evidence of the variation of mean linear velocity with absolute magnitude.

Form of the luminosity function.—For $H > 7.5$ the coefficient b is so small that the mean parallax is nearly independent of m. This result agrees with *Contribution* No. 273 and strengthens the conclusions therein stated relative to stars of very low luminosity. The frequencies of the absolute magnitudes continue to increase to $M = 14$ at least.

Theoretical considerations show that the geometrical mean parallax of stars of magnitude m and proper motion μ may be expressed in the form[2]

$$\log \underline{\pi} - \log \mu = a + bH + cm + dH^2 + \cdots, \qquad (1)$$

where

$$H = m + 5 \log \mu .$$

For a range of 10 or 12 units in H, equation (1) may be replaced by a linear relation in H; for stars within one or two hundred parsecs of the sun, where the space density is nearly constant, the term in m is negligible. The formula then involves but two constants, which may be determined by grouping the observational data according to the single characteristic H. The advantage of such a procedure, especially when the data are scanty, is obvious. Even when the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 438.

[2] *Mt. Wilson Contr.*, No. 273; *Astrophysical Journal*, **59**, 310, 1924.

third constant, c, must be included, a grouping according to H is still to be preferred, for then, since the maximum value of c is only 0.03, the three constants can easily be found by approximations.

Practical tests fully justified these conclusions, but brought to light the important fact that the coefficient of H has different values on either side of the limit $H=7.5$. Thus[1] for 373 stars, $\mu \geqslant 0''.3$, $H<7.5$,

$$\log \underline{\pi} - \log \mu = -0.72 - 0.066\, H\ , \qquad (2)$$

while for 76 stars, $\mu \geqslant 0''.5$, $H>7.5$,

$$\log \underline{\pi} - \log \mu = -1.17 - 0.006\, H\ . \qquad (3)$$

The transition from (2) to (3) probably is not sudden, but for the present it may be regarded as such. These expressions may be compared with formulae giving arithmetical mean parallaxes with the aid of the relation[2]

$$\log \bar{\pi} - \log \underline{\pi} = +0.065\ . \qquad (3a)$$

Equation (2) agrees closely with the formula of Kapteyn and van Rhijn[3] and with the analytical expression based on the distribution functions for density, tangential velocity, and absolute magnitude. Equation (3), however, does not agree with any previously known relation or with the analytical developments. It indicates that for $H>7.5$ the mean parallax is nearly independent of m and that, on the average, $\bar{\pi}=\mu/15$.

The difference between (2) and (3) is, however, real. The formula of Kapteyn and van Rhijn depends on data for which in general $H<7.5$, and is not to be extrapolated beyond this limit. In deriving the luminosity function, however, it was used without restriction, and thus necessarily led to a Gaussian form for this function. Equation (3) refers mainly to stars having absolute magnitudes which lie beyond the maximum of the Gaussian function, on a part of the curve which had not previously been put to observational test. That (3) differs from (2) means, therefore, that the luminosity-curve

[1] *Ibid.*, equations (46) and (47). [2] *Ibid.*, Table I, last column.

[3] *Mt. Wilson Contr.*, No. 188, equation (8); *Astrophysical Journal*, **52**, 23, 1920.

is not Gaussian and that the frequencies continue to increase for several magnitudes beyond the maximum of the Gaussian curve.[1]

Since equation (3) showed the necessity of revising the luminosity function and was itself the basis for calculating the frequencies in the region of low luminosity, it is of special importance. Any confirmation of its form or any revision of its coefficients is therefore to be welcomed, for it summarizes all that we know of the frequencies of intrinsically faint stars.

The results by van Maanen and Willis in *Contribution* No. 412, 1930, are in this respect a valuable contribution. Examination of about 60,000 faint stars in 42 Selected Areas revealed 122 stars having $\mu > 0''.04$. For 113 of these stars ($m > 11$, $\mu < 0''.5$) arranged in 9 groups, they calculated arithmetical mean parallaxes from the τ components of the proper motion and an assumed value of 50 km/sec. for the linear velocity. Van Maanen and Willis then found that the results could be well represented by the formula

$$\log \bar{\pi} = -1.207 - 0.011m + 0.862 \log \mu \,. \qquad (4)$$

They also found that this formula represents the mean parallaxes of 282 stars of the fourteenth magnitude calculated by them from proper motions measured by Ross and by Wolf. Van Maanen and Willis have commented especially on the unimportance of the term in m, the status of which is the more noteworthy since the probable error of its coefficient is ± 0.008.

With the aid of $\log \mu = 0.2(H - m)$, equation (4) may be written in the form

$$\log \bar{\pi} - \log \mu = -1.207 - 0.028\, H + 0.017m \,. \qquad (5)$$

For all of van Maanen and Willis' data, $H > 7.5$; hence (5) is to be compared with the relation

$$\log \bar{\pi} - \log \mu = -1.105 - 0.006H \,, \qquad (6)$$

found by combining (3) and (3*a*).

The uncertainty of the coefficient of H in (5) and (6) is about ± 0.015; the probable error of the term in m is ± 0.018. Aside from

[1] *Mt. Wilson Contr.*, No. 273, Fig. 2; *Astrophysical Journal*, **59**, 335, 1924.

the constant term, which in (5) depends on the assumed linear velocity, the two formulae therefore agree within the limits of uncertainty.

This result is of interest, not merely because the data involved in (5) are wholly new, but because the range in m and in μ to which the very simple form (6) is applicable has been greatly extended. The values of H are much the same in the two cases; but for (6) all the proper motions are $>0''.5$, while for (5) all are $<0''.5$; in fact, for 105 of the 113 stars used, $\mu<0''.3$. Further, nearly all the stars entering into (6) are brighter than magnitude 11, while those used for (5) are all fainter than 11, and 80 per cent of them are fainter than 14.

TABLE I

REPRESENTATION BY MEAN-PARALLAX FORMULA

1	2	3	4	5	6	7	8	9	10	11	12	13
Group	m	μ	$\log \bar{\pi}_\nu$	No.	H	O−C$_1$	$\bar{M}$	$\Delta \log \bar{\pi}_\nu$	O−C$_2$	O−C$_3$	Obs. $\bar{\pi}$	Cal. $\bar{\pi}$
1...	13.19	0".070	−2.310	6	7.4	+0.089	6.5	+0.024	+0.039	+0.085	0".0060	0".0055
2...	16.06	.075	−2.347	17	10.4	+ .052	9.0	.000	.000	+ .012	.0052	.0052
3...	17.86	.073	−2.347	20	12.2	+ .082	10.7	− .007	+ .037	+ .028	.0052	.0047
4...	12.20	.162	−2.022	14	8.2	+ .020	6.9	+ .019	− .029	+ .008	.0116	.0124
5...	15.76	.163	−2.046	29	11.8	+ .030	10.2	− .005	− .017	− .020	.0104	.0108
6...	18.04	.137	−2.260	19	13.7	(− .090)	11.3	− .010	− .127	(− .153)	.0063	.0084
7...	12.86	.348	−1.697	4	10.6	+ .037	9.0	.000	− .013	− .003	.0234	.0242
8...	15.28	.344	−1 750	2	13.0	+ .013	11.2	− .010	− .029	− .044	.0203	.0217
9...	17.34	.466	−1.449	2	15.7	+ .210	14.6	− .025	+ .172	+ .124	.0392	.0264
10...	14.29	.075	−2.444	13	8.7	− .062	2.4	+ .130	+ .003	+ .035	.0057	.0056
11...	14.44	.198	−2.018	126	10.9	− .036	4.4	+ .074	− .010	− .004	.0133	.0136
12...	14.49	.383	−1.710	80	12.4	+ .001	5.8	+ .041	+ .005	− .006	.0250	.0247
13...	14.53	0.664	−1.454	51	13.6	+ .030	7.0	+ .016	+ .018	− .005	.0426	.0408
14...	14.33	1.941	−1.189	12	15.8	(−0.149)	8.1	+0.005	(−0.156)	(−0.205)	0.0764	0.1094

It seems worth while, therefore, to determine the coefficient of H by using the stars of Ross and of Wolf, as well as those of van Maanen and Willis. By disregarding the term in m, simple graphical solutions can be made, which later may be examined for any dependence on m. The essential data collected by van Maanen and Willis in Tables VII and XIII of their paper are repeated in the first five columns of Table I of this paper. Groups 1–9 are van Maanen and Willis' own stars (series I); groups 10–14 those of Ross and of Wolf (series II).

A plot of $D=\log \bar{\pi}_\nu-\log \mu$ against H brought to light at once a systematic difference between series I and II. For I, D decreases with increasing H. For groups 10–13 of II, it increases steadily

and smoothly; the low-weight group 14 is, however, discordant. A provisional formula,

$$\log \bar{\pi} - \log \mu = -1.17 - 0.010H \,, \tag{7}$$

gave the residuals $O-C_1$ in column 7, Table I. The systematic difference between series I and II appears partly in the progression in the residuals for II and partly in the zero-point. These residuals are clearly correlated with the absolute magnitudes in column 8, which range over an interval of 12 magnitudes. The correlation undoubtedly arises from the use of a constant linear velocity in calculating the values of $\log \bar{\pi}_v$; a value of 50 km/sec. may be appropriate for stars of very low luminosity, but it can scarcely apply to the brighter stars.

Plotted against $\overline{M}$, the values of $O-C_1$ define a reasonably smooth curve, whose ordinates may be regarded as corrections to the values of $\log \bar{\pi}_v$ which compensate for the use of a constant linear velocity. The only discordances are groups 6 and 14, which fall far out of line and have been disregarded. The values of $\overline{M}$ were found from

$$\overline{M} = m + 5 + 5 \log \underline{\pi} = m + 5 + 5 \log \bar{\pi} - 0.32 \,; \tag{8}$$

but since $\overline{M}$ is affected by changes in $\log \bar{\pi}_v$, a second approximation was required to obtain the adopted values in column 8. The mean absolute magnitude for series I, to which the constant velocity of 50 km/sec. has been assigned, is about 9.0. The values of $\log \bar{\pi}_v$ were therefore all reduced to this value of $\overline{M}$ by corrections read from the smooth curve (second column, Table II, and column 9, Table I).

A revision of the values of D and a replotting of the results against H then gave a consistent series of points in which the only discordances were again groups 6 and 14. With weights proportional to the number of stars for all groups except 14, which was disregarded, the corrected formula is

$$\log \bar{\pi} - \log \mu = -1.040 - 0.0175H \,. \tag{9}$$

The corresponding residuals $O-C_2$ are in column 10, Table I. For series I these are very similar to the residuals given by van

Maanen and Willis' three-constant formula; for series II there is naturally an improvement because of allowance for the dependence of linear velocity on absolute magnitude. The values of $O-C_2$ show no correlation with the apparent magnitudes; hence there seems to be no necessity for including in the formula a term depending on m.

The scale of distance underlying equation (9) and the corrected values of log $\bar{\pi}_v$ on which the formula is based is determined by the

TABLE II

REVISION OF PARALLAXES

$\bar{M}$	$\Delta \log \bar{\pi}_v$	$\bar{V}_c$	$\bar{M}$	$\Delta \log \bar{\pi}_v$	$\bar{V}_c$
2.....	+0.145	30.7	8.....	+0.006	42.3
3.....	+ .115	32.9	9.....	.000	42.8
4.....	+ .086	35.2	10.....	− .005	43.4
5.....	+ .061	37.2	11.....	− .009	43.8
6.....	+ .037	39.4	12.....	− .013	44.2
7.....	+0.016	41.3	13.....	−0.017	44.6

velocity of 50 km/sec. assigned to stars of absolute magnitude 9. Since the parallaxes were derived from

$$\bar{\pi}_v = 4.74\,\bar{\tau}/50\,, \qquad (10)$$

the velocities for other values of M which are implied by the corrections (log C) given in the second column of Table II are equal to $50/C$ km/sec.

The scale thus adopted—in other words, the zero-point for equation (9)—may be tested by comparing the revised values of log $\bar{\pi}_v$ with equation (6), which was derived from 76 stars of measured parallax. The weighted mean difference (groups 6 and 14 excluded) is

$$\text{Eq. (6)} - \text{Corr. log } \bar{\pi}_v = +0.067\,, \qquad (11)$$

with residuals $O-C_3$ as in column 11, Table I. These residuals are but little inferior to those in the preceding column given by equation (9); the large percentage difference in the coefficients of H in (6) and (9) therefore has had little effect on the comparison. Further, since the range in H is nearly the same for the data entering into the two formulae, the difference (11) may be used provisionally to revise the

scale of distance and to correct the assumed value of the linear velocity.[1]

The final revision of the mean parallaxes is made by applying the difference (11) and the corrections for dependence on M in column 9, Table I, to the values of log $\bar{\pi}_\nu$ in column 4. The resulting parallaxes are in column 12. The mean parallax formula representing the adopted parallaxes, which is obtained by applying the difference (11) to equation (9), is

$$\log \bar{\pi} - \log \mu = -0.973 - 0.0175H\ , \quad (H > 7.5)\ . \tag{12}$$

The values of $\bar{\pi}$ computed with this formula are in the last column of Table I. The residuals in the logarithms are still the values $O-C_2$ in column 10.

From equation (10) the logarithm of the corrected linear velocity is

$$\log \bar{V}_c = \log 50 - (\log \bar{\pi}_\nu - \log \bar{\pi})\ ,$$

where the parenthesis represents the total correction to log $\bar{\pi}_\nu$, i e., the zero-point difference (11) and the correction depending on M given in Table II. The results for $\bar{V}_c$ in the third column of Table II refer specifically to the stars used in this discussion, and not to all stars having the corresponding values of M. Since proper motions $<0''.04$ were not used, the selection in the data favors stars of high speed. The values in Table II average about 10 km/sec. higher than those corresponding to a representative selection of proper motions.[2]

[1] The comparison in the text is primarily a test of equation (6). A direct comparison of (9) with the 76 stars used in *Contribution* No. 273 to determine (6) gives +0.055 for the zero-point correction to (9). This difference should in general be more reliable than that of (11). The apparent magnitudes of *Contribution* No. 273, are, however, supposed to be on the visual system, while those used here are photographic values. Allowance for this difference would increase the zero-point correction by a small but uncertain amount. For the present, (11) may be accepted as a provisional reduction to absolute parallax.

[2] *Mt. Wilson Contr.*, No. 272, Table VII, third column; *Astrophysical Journal*, **59**, 304, 1924. The mean peculiar radial velocities of this table, which were derived from the data of Adams, Strömberg, and Joy, should be directly comparable with the true mean velocities (that is, velocities freed from the influence of selection) derived from τ components.

Equation (12) may be regarded as established for the interval $H = 7.5$ to 14.5. The proper motions involved cover the entire range above $0''.04$. The apparent magnitudes range from the upper limit possible, near 5 or 6, to a lower limit at about 19. The uncertainty in the coefficient of H is about 0.005; that in the zero-point corresponds to about 4 or 5 per cent in the parallaxes. The zero-point error might have been reduced somewhat by including the data on measured parallaxes that have accumulated since equation (6) was established; but other errors would still have remained, especially those arising from the very large uncertainty in the apparent magnitudes of many of the stars.

As it stands, equation (12) is an excellent general confirmation of (6). Hence, the conclusions of *Contribution* No. 273 in respect to the form of the luminosity function in the region of very low luminosities are also confirmed: In particular, the frequencies of absolute magnitudes must continue to increase to at least the fourteenth magnitude. The corresponding space density of stars near the sun (stars brighter than $M = 14$) is of the order of one star per 10 cubic parsecs, about twice the value resulting from the Gaussian form of the luminosity function.

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TEMPERATURE CLASSIFICATION OF THE SPECTRA OF YTTERBIUM AND LUTECIUM[1]

By ARTHUR S. KING

ABSTRACT

The spectra of ytterbium and lutecium produced by the electric furnace, arc, and spark have been examined between λ 2950 and λ 6800 with regard to the segregation of neutral and enhanced lines, temperature classification, the occurrence of hyperfine structure, and other distinctive features. The lines listed include 399 for ytterbium and 103 for lutecium.

Widely separated energy-levels in the spectrum of neutral ytterbium are indicated by the very different response to the excitations used. About 70 per cent of the ytterbium lines listed belong to the ionized atom; these were placed in three groups according to the degree of enhancement in the spark.

Neutral lutecium lines are in general strong in the furnace as compared with the arc, while the enhanced lines, with one exception, λ 3507, require high excitation. The persistence of this line in the furnace indicates a low ionization potential for lutecium.

Hyperfine structure is present in about half of the lutecium lines examined, resolution of the components being obtained in some cases. Three-component lines seem to be of most frequent occurrence, and a tendency toward wider spacing in the patterns is found for lines appearing at low excitation.

This investigation follows the plan of previous studies of rare-earth spectra,[2] the purpose being to obtain the data that the electric furnace can supply on the temperatures required for the initial appearance of line-groups, as well as on the variation of line-intensities with temperature change. These data are supplemented by a comparison with the effects obtained, especially for the lines of the ionized atom, by means of the higher excitations of the arc and spark.

In the present study, which is for the spectral range from λ 2950 to λ 6800, the procedure was the usual one of observing the furnace spectrum at three temperature stages, in this case 2600°, 2300°, and 2100° C, in order to determine the approximate temperature at which successive groups of lines appeared and their rate of change at higher temperatures. The temperature classification of the stronger neutral lines was obtained in this way, and at the higher furnace temperature some of the low-excitation lines of the singly ionized

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 439.

[2] A. S. King, *Mt. Wilson Contr.*, Nos. 368, 414; *Astrophysical Journal*, **68**, 194, 1928; **72**, 221, 1930.

spectrum appeared. A comparison of arc and spark spectra was made as a means of selecting the ionized lines; and, in the case of ytterbium, many faint lines were brought out in the arc which did not appear in the furnace spectrum. Such lines go into class V. Lines appearing first at high and medium temperatures are of classes IV and III, respectively, while the low-temperature lines are assigned to class II or I, according to their persistence at low temperature.

The identification of lines, especially those of ytterbium, has been a matter of considerable difficulty on account of the close chemical affinity of ytterbium with thulium. Three samples, prepared by G. Urbain of Paris and obtained from Professor G. Eberhard of Potsdam, were used in this investigation. Two were mixtures of ytterbium and lutecium with some thulium present, while the third was much richer in thulium. By means of the relative abundance in the three samples, as indicated by distinctive lines, the thulium lines were sorted out as completely as possible, but some probably remain in the ytterbium table. Many lines in the ytterbium list of Exner and Haschek[1] evidently belong to thulium. Some lines given by J. M. Eder[2] as possibly due to thulium have been included in my ytterbium table, because of their strength on spectrograms on which thulium lines in general were weak. The identity of lutecium lines is better established. In preparing my lists for these spectra I have been greatly assisted by unpublished lists, kindly placed at my disposal by Dr. W. F. Meggers, which permit a comparison with the identifications made in an independent investigation.

The spectrograms, for wave-lengths less than 4700, were made in the second order of the 15-foot concave grating; for the red end of the spectrum the first order was used. The furnace was charged with a small quantity of the oxide, held in a graphite combustion boat. For the arc and spark, silver rods were used, the oxide being mixed with silver filings placed in a small cavity in the lower (positive) electrode.

EXPLANATION OF THE TABLES

Wave-lengths on the international system are given in the first column of Tables I and II, and I am greatly indebted to Dr. Meggers

[1] *Spektren der Elemente*, **2**, 1911.

[2] *Sitzungsberichte der Akademie der Wissenschaften in Wien*, IIa, **124**, 707, 1915.

for allowing free use of his measurements for the spectra of both ytterbium and lutecium. The wave-lengths in Table I are for the most part those of Meggers, some are from Eder, some converted from Exner and Haschek, and a few lines, usually components of close doublets, are from measures by the writer. The lutecium wave-lengths in Table II are by Meggers, with some measurements, chiefly for hyperfine lines, by the writer.

The intensities in the second and third columns are for the arc spectrum and for that of the furnace at high temperature. The intensities at medium and low temperatures are not entered, but were taken into account in assigning the classes in the final column. The enhanced lines have "E" after the class number, and for the ytterbium lines subscripts are added to indicate the degree of enhancement in the spark spectrum. "A" following the class number for many lutecium lines in Table II denotes a relatively high intensity in the furnace as compared with the arc. An asterisk (*) after the wave-length refers to a note at the end of the table.

In Table II an Arabic figure after the class number indicates that the lutecium line in question clearly has hyperfine structure, and gives (purely tentatively, since few lines are fully resolved) the number of hyperfine components. The patterns appear usually to have either three or four components, with the strongest on the short wave-length side. Features in which the line differs from this prevailing type are mentioned in the notes at the end of the table.

TABLE I

TEMPERATURE CLASSIFICATION OF YTTERBIUM LINES

λ	Intensity		Class	λ	Intensity		Class
	Arc	Furnace			Arc	Furnace	
2935.09	4	.....	V E_2	3044.83	1	.....	V E_2
2937.18	2	.....	V E_2	3046.47	4	.....	V E_2
2938.20	1	.....	V E_2	3047.05	2	.....	V E_2
2939.51	2	.....	V E_2	3055.16	2	.....	V E_1
2940.52	4	.....	V E_2	3063.12	3	.....	V E_1
2942.03	2	.....	V E_2	3063.68*	3	.....	V E_1
2942.82	1	.....	V E_2	3064.92	1	.....	V E_2
2945.91	15	.....	V E_2	3065.03	10	.....	V E_2
2946.29	4	.....	V E_2	3073.68	1	.....	V E_1
2946.76	1	.....	V E_3	3076.04	1	.....	V E_1
2950.32	2	.....	V E_2	3085.82	1	.....	V E_1
2951.02	1	.....	V E_2	3089.10	10	.....	V E_1
2955.31	3	.....	V E_1	3092.54	.....	.....	V E_3
2960.85	2	.....	V E_1	3093.87	8	.....	V E_2
2962.51	5	.....	V E_2	3095.40	1	.....	V
2963.44	5	.....	V E_2	3101.35	4	.....	V E_2
2964.41	2	.....	V E_2	3102.07	3	.....	V E_2
2964.74	10	.....	V E_2	3107.75*	6	.....	V E_2
2970.57	150	.....	V E_1	3107.91*	30	.....	V E_1
2970.83	2	.....	V E_3	3115.32	8	.....	V E_1
2979.86	1	.....	V E_1	3116.07	1	.....	V E_2
2982.50	4	.....	V E_1	3116.51	1	.....	V E_2
2982.66	1	.....	V E_2	3116.71	4	.....	V E_2
2983.98	15	.....	V E_2	3117.81	20	.....	V E_2
2985.09	6	.....	V E_2	3125.48	1	.....	V E_1
2985.86	2	.....	V E_2	3126.09	1	.....	V E_3
2990.36	2	.....	V E_2	3132.63	1	.....	V E_1
2991.86	4	.....	V E_2	3136.74	4	.....	V E_2
2993.93	2	.....	V E_2	3140.92	15	.....	V E_2
2994.80	10	.....	V E_2	3141.72	4	.....	V E_2
2995.85	2	.....	V E_2	3145.06	5	.....	V E_2
2998.03	1	.....	V E_3	3145.55	3	.....	V
3000.46	5	.....	V E_2	3149.00	2	.....	V E_1
3002.61	2	.....	V E_2	3153.17	2	.....	V E_2
3005.76	40	.....	V E_2	3153.85	6	.....	V E_3
3009.38	6	.....	V E_2	3155.17	3	.....	V E_2
3010.61	4	.....	V E_2	3158.29	2	.....	V E_2
3014.47	4	.....	V E_2	3162.29	1	.....	V
3017.55	10	.....	V E_2	3163.78	6	.....	V E_2
3022.45	1	.....	V E_2	3165.20	4	.....	V E_2
3026.67	10	.....	V E_2	3169.04	10	.....	V E_2
3029.25	2	.....	V	3171.19	2	.....	V E_1
3029.54	1	.....	V E_3	3173.77	1	.....	V E_2
3031.11	100	.....	V E_1	3175.74	2	.....	V E_2
3033.86	1	.....	V E_2	3180.90	8	.....	V E_2
3034.63	4	.....	V E_1	3190.81	1	.....	V E_1
3036.82	1	.....	V E_1	3191.41	1	.....	V E_3
3038.00	2	.....	V E_1	3192.87	50	.....	V E_2
3039.66	4	.....	V E_2	3194.24	2	.....	V E_2
3042.65	6	.....	V E_2	3194.76	2	.....	V E_2
3044.00	2	.....	V E_1	3198.65	8	.....	V E_2

TABLE I—*Continued*

λ	Intensity		Class	λ	Intensity		Class
	Arc	Furnace			Arc	Furnace	
3201.15	25		V E_2	3404.09	4		V E_2
3210.11	1		V E_2	3411.27	1		V
3217.17	8		V E_2	3412.48	4		V
3218.32	5		V E_2	3416.88	1		V E_2
3225.86	5		V E_2	3417.04	1		V
3228.61	1		V E_3	3418.40	8		V
3229.84	1		V E_1	3419.62	1		V E_3
3231.97	2		V E_2	3420.37	2		V
3236.15	4		V E_2	3426.05	25	1—	V
3239.20	3		V E_2	3428.46	6		V E_2
3239.61	2		V	3431.15*	15		V
3259.10	4		V E_2	3431.37	1		V
3261.50	10		V E_1	3436.44	4		V E_2
3275.81	2		V E_2	3438.72	8		V E_2
3289.36*	1000	50	IV E_1	3438.82	15		V E_2
3297.85	1		V E_2	3443.61	1		V
3304.57	4		V E_2	3446.89*	3		V E_1
3304.75	4		V E_2	3452.38	4		V
3305.24	6		V	3454.06	60		V E_2
3305.72	15		V E_1	3458.27	8		V E_2
3306.78	4		V E_1	3459.68	1		V
3307.99	2		V	3460.27	15		V
3309.37	3		V E_2	3464.19*	8		V
3315.09	1		V	3464.37*	500	100*R*	I
3316.52	1		V E_1	3467.04	2		V E_1
3319.02	2		V	3472.33	2		V
3319.17	6		V E_2	3474.12	1		V
3319.41	12		V	3474.80	2		V E_2
3324.17	2		V E_2	3475.43	6		V E_2
3324.45	1		V E_1	3476.29	100	20	III
3333.05	4		V E_2	3478.82	80	1—	V E_2
3337.17	8		V	3480.43*	12		V E_2
3342.93*	6	10	III	3485.74	5		V E_2
3343.06*	3		V E_2	3487.06*	12		V E_2
3346.51	4		V	3488.80	2		V E_1
3347.53	5		V E_2	3507.82	5		V E_2
3352.48	5		V	3515.86	2		V E_2
3355.88	2		V E_1	3518.15	3		V E_2
3359.24	2		V	3520.27	20		V E_2
3362.49	3		V	3549.81	15		V E_2
3363.67	2		V	3552.34	2		V E_2
3365.96	1		V E_2	3560.32	30		V E_1
3375.48	15		V E_2	3560.70	20		V E_2
3379.77*	3?		V E_2	3567.11	3		V E_2
3387.49	6		V E_1	3570.55	8		V E_2
3390.24*	2		V E_3	3585.46	30		V E_1
3391.10	3		V E_2	3606.46	8		V E_2
3394.43	2		V E_1	3611.30	2		V E_2
3396.31	1		V E_1	3619.80	15		V E_2
3398.67*	1		V E_3	3634.56	2		V
3401.01	4		V E_2	3637.74	15		V E_2
3402.28	2		V E_1	3648.11	2		V

TABLE I—*Continued*

λ	Intensity		Class	λ	Intensity		Class
	Arc	Furnace			Arc	Furnace	
3655.73	6	.....	V	4257.75	1	.....	V
3669.71	20	.....	V E_1	4277.73	6	1	IV
3670.67	2	.....	V E_2	4300.97	2	.....	V
3675.07	15	.....	V E_2	4316.97	10	.....	V E_1
3690.57	3	.....	V E_2	4322.21	2	.....	V E_2
3691.47	1	.....	V E_2	4339.11	1	.....	V E_2
3694.20*	1000	20	IV E_1	4370.81	2	.....	V E_2
3698.57	4	.....	V E_2	4393.77*	5	1	IV
3708.66	2	.....	V E_1	4402.30	1	.....	V E_2
3722.29	2	.....	V E_2	4427.44	2	.....	V
3724.20	4	.....	V E_2	4430.23	3	1	IV
3730.40	1	.....	V E_2	4439.21	30	15	III
3734.70	10	4	IV	4482.45	4	1	IV
3749.66	3	.....	V E_2	4493.98	1	.....	V E_2
3770.10	20	6	IV	4515.17	12	.....	V E_2
3773.44	2	.....	V E_2	4529.89	4	3	III
3782.53	3	.....	V E_2	4553.59	4	.....	V E_2
3791.74	4	4	IV	4564.00	6	8	III
3807.52	2	.....	V E_2	4576.23	80	40	III
3814.22	1	.....	V E_2	4582.37	20	10	III
3816.21	2	.....	V E_2	4589.24	4	.....	V
3872.85*	10	?	IV ?	4590.84	8	2?	IV ?
3900.86*	12	6	III	4598.38	5	.....	V E_2
3904.82	2	.....	V E_3	4644.55	5	4	III
3911.26	2	.....	V	4650.07	4	?	V
3934.30	1	.....	V E_2	4670.59	3	.....	V E_1
3938.26	1	.....	V E_2	4683.82	8	.....	V E_2
3946.93	1—	.....	V E_2	4684.27	8	?	IV ?
3987.99	1500	1000*R*	I	4704.90	2	.....	V
3990.89	40	8	IV	4718.67	3	4?	III
4007.36	2	.....	V	4726.10	60	.....	V E_1
4040.08	1	.....	V	4732.03	1	.....	V E_1
4052.28	8	5	III	4751.79	1	.....	V
4077.27	2	.....	V E_2	4752.93	3	.....	V E_1
4089.69	20	6	III	4770.18	1	.....	V
4091.50	1	.....	V E_1	4781.90	20	6	II
4113.06	1	.....	V E_2	4784.53	1	.....	V
4119.40	2	.....	V E_2	4786.62	80	1	V E_1
4122.87	2	.....	V E_1	4816.41*	4	3	III
4135.10	8	.....	V E_2	4818.38	3	.....	V E_2
4149.07	30	8	III	4820.25	10	.....	V E_2
4170.11	2	.....	V E_1	4836.95	8	.....	V E_2
4174.58	4	3	III	4837.46	3	2	IV
4180.83	40	.....	V E_1	4847.4*	2	.....	V E_2
4190.31	3	.....	V E_2	4848.44	1	.....	V E_2
4218.57*	3	.....	V E_2	4851.16	4	.....	V E_2
4218.69*	5	6	III	4894.63	2	2	III
4231.99	6	10	III	4912.36	1	1	IV
4234.53	1	.....	V E_2	4935.50	600	20	II
4252.51	3	.....	V E_2	4937.23	4	.....	V E_2
4255.78	2	.....	V E_2	4944.08	2	.....	V E_2
4257.64	2	.....	V E_2	4966.90	20	6	II

TABLE I—*Continued*

λ	Intensity		Class	λ	Intensity		Class
	Arc	Furnace			Arc	Furnace	
5009.53*	6	.....	V E_2	5531.1*	1	.....	V
5021.14	1—	.....	V E_2	5539.11*	15	10?	III
5067.34	1	.....	V E_2	5547.18	1	.....	V E_1
5067.83	1	.....	V	5556.49	1000	500*R*	I
5069.17	3	.....	V	5562.08	4	.....	V
5074.34	30	4	III	5568.10	1	.....	V
5076.77	2	.....	V	5580.83	1	.....	V E_3
5136.00	3	.....	V E_1	5586.36	1	2	IV A
5147.03	2	.....	V E_1	5588.47	15	.....	V E_2
5173.13	1	.....	V E_2	5637.87	1	.....	V E_1
5184.18	5	.....	V E_2	5652.01*	20	.....	V E_2
5196.11	5	2	IV	5686.57	1	.....	V E_2
5200.56	4	.....	V E_2	5700.2*	1	.....	V
5211.59	8	5	III	5713.76	1—	.....	V E_2
5227.3*	2	1	IV	5720.01	40	15	III
5228.20	1	.....	V	5728.92	1—	.....	V E_2
5240.52	6	.....	V E_2	5730.03	1	.....	V E_3
5243.8*	1	.....	V E_1	5749.92	1—	.....	V E_2
5244.12	10	5	IV	5767.24	1—	.....	V E_3
5257.50	12	.....	V E_2	5771.67	10	.....	V E_2
5271.6*	1	.....	V	5803.46	1	.....	V
5277.08	60	8	III	5819.45	3	.....	V E_3
5279.56	15	.....	V E_2	5834.00	2	.....	V
5300.94	4	.....	V E_2	5837.16	30	.....	V E_2
5335.16	200	.....	V E_2	5854.49	2	4	III A
5345.69	5	.....	V E_2	5897.25	3	.....	V E_3
5345.82	3	.....	V E_2	5898.80	1—	.....	V E_3
5347.22	40	.....	V E_2	5908.37	4	.....	V E_2
5351.33*	2	5?	III A ?	5935.37	1	.....	V E_3
5352.96	150	.....	V E_2	5946.03	2	.....	V E_3
5358.65	15	.....	V E_2	5991.51*	20	.....	V E_2
5363.67	3	4	IV	6052.88	2	.....	V E_2
5389.87	4	.....	V E_2	6152.59	20	.....	V E_2
5390.77	2	.....	V	6246.97	8	.....	V E_2
5399.76	2	.....	V E_2	6260.82	1	.....	V E_3
5426.90	1	.....	V E_2	6274.80	40	.....	V E_2
5432.74	3	.....	V E_3	6308.16	1—	.....	V E_2
5449.29	20	.....	V E_2	6400.40	10	3	IV
5454.04*	2	.....	V E_3	6417.97	4	.....	V
5455.08	2	.....	V E_2	6432.75	4	.....	V E_2
5478.53	6	.....	V E_2	6489.10	200	200	II
5481.95	10	5	III	6667.82	30	15	III
5501.65	1	.....	V	6727.53	1—	.....	V E_2
5505.51	5	?	V ?	6799.58	150	60	II
5524.57	1	.....	V				

NOTES TO TABLE I

λ	
3063.68	Double.
3107.75	Measured by writer. Equal strength in arc and spark. λ 3107.91 winged to red in spark.
3107.91	
3289.36	Reversed in spark.
3342.93	Measured by writer. Other observers have measured blend. Furnace intensity of λ 3342.93 exceptional in this region, but no other identification found.
3343.06	
3379.77	In wing of *Ag* line.
3390.24	Wide in spark.
3398.67	Wide in spark.
3431.15	Faint *Yb* II on red side.
3446.89	Blend *Tu* I.
3464.19	Measured by writer.
3464.37	Measured by writer.
3480.43	Origin uncertain.
3487.06	May be *Tu*.
3694.20	Reversed in spark.
3872.85	In *CN* band.
3900.86	Blend *Tu*.
4218.57	Measured by writer.
4218.69	Measured by writer.
4393.77	Diffuse. May be double.
4816.41	Double.
4847.4	Double. Measured by writer.
5009.53	*Yb* I on violet side.
5227.3	Measured by writer.
5243.8	Measured by writer.
5271.6	Measured by writer.
5351.33	Furnace line may be partly *C*.
5454.04	Diffuse in spark.
5531.1	Measured by writer.
5539.11	Blend *C* at high temperature.
5652.01	Probably blend *Tu* I.
5700.2	Measured by writer.
5991.51	Measured by writer.

TABLE II

TEMPERATURE CLASSIFICATION OF LUTECIUM LINES

λ	INTENSITY		CLASS	λ	INTENSITY		CLASS
	Arc	Furnace			Arc	Furnace	
2951.69	10	.....	V E	4054.47	8	20	III A
2963.33	50	.....	V E, 3	4112.68	3	12 ?	III A, 3
2969.81	30	.....	V E, 3	4122.49*	6	.....	V, 3
2989.27	20	.....	V	4124.73	100	30	III
2995.84	1	.....	V E	4154.08	10	10	III
3020.54	40	.....	V E, 3	4184.26	50	.....	V E, 4
3056.72*	30	.....	V E, 3	4277.50	4	1 ?	IV ?
3077.60	100	.....	V E, 3	4281.03	6	10	V, 3
3080.11	4	5	IV	4295.950*	5	10	III A
3081.46	·50	8	III	4296.114*	3	8	III A
3118.43	30	4	III	4309.57*	6	30	III A, 4
3171.36	20	4	III	4332.72	2	.....	V
3191.81	3	.....	V	4420.96	2	.....	V, 3
3198.11*	30	.....	V E, 3	4430.47	2	.....	V
3254.31	50	.....	V E, 3	4450.81	4	10	III A, 3
3265.62	5	.....	V	4480.15	1	5	III A, 4
3278.97	30	8	III	4498.85	3	30	III A, 3
3280.50*	?	4	IV	4518.58	200	300	II, 4
3281.73	40	20	III	4605.39*	3	.....	V, 3
3312.11	100	60r	II	4610.3*	5	.....	V, 3
3359.57	80	40	III	4643.30	1	10	III A, 4
3376.51	60	50	III	4645.48	5	.....	V, 3
3385.49	8	20	III A	4648.22*	5	.....	V, 3
3391.56	4 ?	4	III	4648.83*	5	.....	V, 3
3396.82	30	30	III	4658.03	40	150	III A, 4
3397.06*	30 ?	1	V E, 3	4659.04	4	.....	V, 3
	20 ?	.....		4675.29*	5	.....	V
	15 ?	.....		4785.43	10	.....	V E, 3
3472.48	60	2	V E	4810.52	3	.....	V
3507* ·31	50	20	III E, 3	4815.05	4	15	III A, 3
3507* ·39	30	12		4839.43*	10	.....	V E, 3
3507* ·47	20	8		4839.62			
3508.41	20	40	III A	4839.79			
3546.39	3	6	III A	4888.14	2	.....	V
3554.43*	80	.....	V E, 3	4904.89	20	20	III, 3
3567.84	100	40	III	4942.33	6	15	III A, 3
3596.33	2	4	IV A, 3	4994.14*	50	.....	V E, 4
3620.31	1	5	IV A	5001.15	50	30	III, 3
3623.98*	15	.....	V E	5135.10	50	100	III A, 4
3636.25	20	60	III A, 3	5196.61	4	.....	V
3642.35*	3	.....	V, 3	5304.40	2	.....	V
3647.77	100	50	III	5349.12	3	.....	V
3684.31*	?	12	III A	5402.56	20	200	II A, 3
3756.70	4	8	III A	5421.90	6	40	III A, 4
3841.18*	30	20 ?	III	5437.88	2	6	III A, 3
3876* ·53	1	.....	V E, 4	5453.57	1	3	III A, 3
3876* ·57	2	.....		5476.70	200	.....	V E
3876* .63	6	.....		5736.55	25	250	II A, 3
3876* .69	10	.....		5775.39	·2	2 ?	IV ?
3925.30	2	.....	V, 3	5800.59	1	8	III A, 3
3968.464*	?	50	III A, 4	5860.79*	1	6 ?	IV A ?

TABLE II—*Continued*

λ	Intensity		Class	λ	Intensity		Class
	Arc	Furnace			Arc	Furnace	
5983.59*	15	.....	V E, 4	6159.95	4	.....	V E
5983.80	4	.....		6199.67	3	.....	V E, 3
5984.00	5	.....		6221.87	200	.....	V E, 3
5984.15	5	.....		6235.37	1	.....	V E
5997.13	2	.....	IV, 3	6242.35	4	.....	V E
6004.52*	100	150	II	6463.11	100	.....	V E, 3
6055.04	15	200	II A, 3				

NOTES TO TABLE II

λ	
3056.72	Components almost resolved.
3198.11	Partially resolved.
3280.50	Blend *Ag* in arc.
3397.06	Not fully resolved.
3507	Components well resolved. Measured by writer.
3554.43	Components very close.
3623.98	Enhancement small.
3642.35	Measured by writer.
3684.31	Faint in arc. Blend *Ag* ghost.
3841.18	Blend *C* in furnace.
3876	Exceptional in stronger component being toward red. Measured by writer.
3968.464	Masked in arc by *Ca* λ 3968.475. Measured in furnace by writer. Probably faint in arc.
4122.49	May be band head.
4295.950 4296.114	Measured by writer.
4309.57	Components almost resolved.
4605.39	May be band head.
4610.3	May be band head. Measured by writer.
4648.22 4648.83	Blend into wide patch.
4675.29	May be band line.
4839	Components blended. All given by Meggers as stronger in spark.
4994.14	Wide pattern. May have more than four components.
5860.79	Furnace line may be impurity.
5984	Measured by writer. Group may consist of two complex lines, each partially resolved.
6004.52	Narrow. May be complex.

DISCUSSION

Ytterbium.—Through most of the range observed, the ytterbium spectrum shows a strong preponderance of enhanced lines. Judged by their enhancement in the spark and their faintness in or absence from the furnace, 283 lines belong to the ionized atoms; a considerable number of fainter lines can also be detected in the spark spectrum only. The degree of variation observed in the change from arc to spark in the case of different ionized lines should indicate their relative energy-levels. In Table I, lines with "E_1" following the class number show a small change from arc to spark. As a rule such lines are among the weaker lines; but two of the strongest enhanced lines, λ 3289.36 and λ 3694.20—probably ultimate lines of *Yb* II—are of this character. These lines reverse in a strong spark and are of fair strength in the furnace. A large majority of the lines show a decided strengthening in the spark and are designated "E_2." A much smaller number are very much enhanced in the spark, being absent from or very faint in the arc. The wave-lengths of these lines are usually converted from values in the spark list of Exner and Haschek. Any doubly ionized lines occurring in Table I are in this group, although, in the visible region at least, E_3 lines are probably high-excitation lines of *Yb* II. We find the enhanced lines distributed throughout the spectrum, although for the most part in the ultraviolet, neutral lines being very few below λ 3300.

The neutral ytterbium lines, 116 in number, arise, if we may judge from their varied response to different excitations, from widely separated energy-levels. In general, the ytterbium spectrum would be classed as of high excitation. About half of the neutral lines are absent from the furnace; some of these, however, are scarcely to be expected in the furnace because of their faintness in the arc. Most of the lines appearing in the furnace are of classes III and IV; only eight are in the low-temperature classes I and II, but these are notable as including the very strong lines λλ 3464.37, 3987.99, and 5556.49, all of class I and all reversing in the furnace. At low temperature these three lines remain very strong, and λ 3988, the most favorable for observation, shows the characteristic of a "flame" line in winging out during the early part of the exposure, when the vapor density is considerable, then changing into a strong narrow line.

Lutecium.—Aside from its relatively few lines, the lutecium spectrum presents many features different from those of ytterbium. The neutral spectrum may be characterized in general as of low excitation, many lines being relatively strong in the furnace. The stronger lines usually show no decided change in intensity between 2300° and 2600° and are placed in classes II and III, while pronounced low-temperature lines are lacking. This last feature accords with the absence of even partial reversals at high temperature, except in the case of λ 3312. It was not possible, however, to do full justice to the lines appearing at low temperature, as this would have required more frequent recharging of the furnace during long exposures than was permitted by the small amount of lutecium at disposal.

A partial term-analysis of the lutecium spectrum has been made by Meggers and Scribner,[1] who found regularities among the stronger lines of neutral and of singly and doubly ionized lutecium. It is of interest to compare the observed behavior of lines with this preliminary term-analysis. The assignment of levels for *Lu* I by Meggers and Scribner is as yet tentative, but they found the difference 1993.9 cm^{-1} to recur for fourteen pairs of the stronger lines. Those within the region here studied fall into classes II and III of the furnace classification, with some differences between furnace and arc in the run of their relative intensities. This condition harmonizes with the conclusion of Meggers and Scribner that the stronger *Lu* I lines form a homogeneous group, in which two main levels are involved.

A strong *Lu* I line not previously observed was measured by the writer at λ 3968.464. That it has not been detected before is due to its faintness in the arc and its difference by only 0.01 A from the H line of calcium, which is likely to occur as an impurity in arc spectra. On the lutecium furnace spectrograms, the K line of calcium, the companion of H, did not appear, thus indicating that H also was absent and that the strong furnace line belonged to lutecium. Dr. Meggers, when informed of this observation, noted that the constant difference 1993.9 cm^{-1} pairs the new line with λ 4518.58, one of the three strong lines not included in his classifica-

[1] *Bureau of Standards Journal of Research*, **5**, 73, 1930.

tion. The computed wave-length is 3968.46 A, in agreement with the measured value.

The stronger lines of ionized lutecium, which appear distinct in the arc spectrum, are included in Table II. Only three of these lines are of low enough excitation to appear in the furnace spectrum, but one of them, λ 3507, is of fair intensity in the furnace as low as 2300°, and is placed in class III E. Meggers and Scribner, who have made good progress in the analysis of *Lu* II, ascribe this line to the lowest level of the ionized atom, the other lines in the range of the present investigation arising from much higher levels. The ease of production of λ 3507 in the furnace indicates that the ionization potential of lutecium is below 6 volts. This estimate is based on the temperatures required for the appearance of enhanced lines in other rare-earth spectra.

Hyperfine structure of lutecium lines, both neutral and ionized, to which attention was called by Meggers and Scribner, is very conspicuous on my spectrograms, which in a few cases show resolution of the structures. The best resolution observed was for λ 3507, mentioned above as from the lowest level of the ionized atom. This line has three components equally spaced, degraded in intensity toward the red. Many unresolved patterns appear to be of the same type as λ 3507, at least to the extent of having the strongest component on the violet edge. The only clear exception noted is λ 3876, for which four components were measured, the strongest being to the red. Several lines very strong in the furnace are of unusual width, and higher resolution will probably show that they have at least four-component structure. Such neutral lines, all with components degraded toward the red, are λλ 3968.46, 4309.57, 4518.58, 4658.03. Taking the spectrum as a whole, we find that about half of the lines in Table II are evidently complex. Lutecium is the fifth in the series of rare earths for which hyperfine structure has been shown to prevail for a large proportion of the lines, and all five are of odd atomic number.

In the ionized spectrum, a certain variation is found in the hyperfine structures of lines arising from the different levels designated by Meggers and Scribner. The low-excitation line λ 3507 ($a^1S_0 - a^3P_1$) shows the best resolution. Next comes a group ($a^3D_1 - a^3P$) consist-

ing of λλ 4839, 5984, 6463, with conspicuously wide patterns, although they require much higher excitation than λ 3507. λλ 4839 and 5984 appear each to have four components, partially resolved in the arc. The lines of the next group ($a^3D_2 - a^3P$), consisting of λλ 3876, 4994, 6222, have probably four components each, but closer than those of the preceding lines. The remaining lines of *Lu* II on my plates appear to have narrower patterns, λ 3397 ($a^3D_1 - a^3F_2$) coming nearest to resolution. The occurrence of wide patterns among the lines of lower energy-level in *Lu* II appears to correspond with the large hyperfine structures found in low-excitation lines of *Lu* I.

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THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG A AND B STARS BRIGHTER THAN THE SIXTH APPARENT MAGNITUDE AS DETERMINED FROM PARALLACTIC MOTIONS[1]

By GUSTAF STRÖMBERG

ABSTRACT

1. The *distributions* of *absolute magnitudes* among A and B stars brighter than apparent visual magnitude 6.0 have been determined from the distributions of parallactic reduced proper motions and of radial velocities, by the method described in *Mt. Wilson Contr.* No. 410. The stars have been grouped according to the six spectral intervals Bo–B2, B3, B5, B8–B9, Ao, and A2–A5. The numbers of proper-motion and radial-velocity stars in each group are shown in Table I.

2. The distributions of absolute magnitudes are shown in Table IV and in Figs. 7–12. The group A2–A5 shows maxima at $M=-3.2$ and $+1.1$, separated by *zero frequencies* around $M=-1$, thus indicating clearly the existence of two distinct groups: *supergiants*, 15 per cent, and *normal* A stars, 85 per cent. The Ao stars show similar maxima at $M=-5.5$ and $+0.6$, with 4 and 96 per cent of the stars in the respective groups. The B8–B9 stars show *supergiants* around $M=-3.5$ and *normal* stars around $M=-0.4$, in the proportion of 15 and 85 per cent, respectively. Among the B5 stars, the *supergiants* appear around $M=-3.8$ and *normal* stars around $M=-1.4$, in the proportion 17 and 83 per cent, respectively. The existence of *normal giants* of absolute magnitude around 0.0 is clearly indicated. The B3 stars show *two groups* of *supergiants*, around $M=-6$ and $M=-4$, and *normal* stars around -1.4, with a relative proportion of 2, 18, and 80 per cent, respectively. *Normal giants* appear to exist around $M=-0.3$. The Bo–B2 stars show *three groups of supergiants* around $M=-8$, -5.5, and -4; stars of the *main sequence* at $M=-2.9$; and *normal giants* at $M=-0.3$. The proportions are 4.5, 2.5, 89 (combination of two groups), and 4 per cent, respectively.

3. The *reality* of the different groups is judged from the continuous run from one spectral group to another, with no overlapping of the groups. Some evidence is given for a *crossing*, at the A stars, in the Russell-Hertzsprung diagram, of the *branch* of the *main sequence* and the *line of normal giants*.

4. The *distributions* of reduced *angular parallactic motions* and of the *corresponding linear motions* are shown in Tables III and II, respectively.

The method for determining the distribution of absolute magnitudes from parallactic motions given in *Mount Wilson Contribution* No. 410[2] has been applied to the A and B stars brighter than the sixth apparent magnitude, with the results given in this paper. The final results for the M, K, G, and F stars have been published in *Contributions* Nos. 411,[3] 418,[4] and 430.[5]

The spectral classification and the apparent magnitudes through-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 440.

[2] *Astrophysical Journal*, **72**, 111, 1930.

[3] *Ibid.*, **72**, 117, 1930.

[4] *Ibid.*, **73**, 40, 1931.

[5] *Ibid.*, **74**, 110, 1931.

out are those of the *H.D. Catalogue*. For known binaries the apparent magnitude of the brighter component has been used; but for the presumably large number of spectroscopic binaries whose differences of magnitude are unknown, the combined apparent magnitude had to be used. For this reason the final absolute magnitudes require a small correction, which, however, can hardly be more than +0.1 mag.

The peculiar motions cannot be used in this analysis, first, because they are much smaller than the parallactic motions, and, even more important, because we cannot determine the radial peculiar motions

TABLE I

Spectrum	Radial Motions			Angular Motions	
	No.	V_0	θ	Total No.	No. $\sin \lambda \geq 0.5$
		km/sec.	km/sec.		
B0–B2	102	24.4	12.0	124	90
B0–B5	446	24.8	10.0	.....	.....
B3	.....	.....	.....	246	188
B5	.....	.....	.....	152	122
B8–B9	243	25.0	9.5	351	287
A0	389	16.1	10.9	416	346
A2–A5	459	16.1	12.5	558	478
cA	19	18.5	13.6	.....	.....

which might be taken as corresponding to the τ-components. These components are more or less perpendicular to the galaxy, and hence are comparable with the peculiar radial velocities in regions around the galactic poles, where the number of A and B stars is too small for a good determination of the distribution of the peculiar radial velocities. On this account Gerasimovič's[1] determination of the luminosity-curves for the early B stars, by a similar analysis based on the τ-components, is affected by large accidental and systematic errors. As in previous studies, Boss's proper motions have been used, and Raymond's[2] corrections have been applied to the proper motions in declination.

Table I gives the numbers of stars in the different groups. Those for radial velocity include stars brighter than the seventh apparent

[1] *Monthly Notices of the Royal Astronomical Society*, **91**, 537, 1931.

[2] *Astronomical Journal*, **36**, 129, 1926.

magnitude. No division according to absolute magnitude was possible, except that the radial velocities of the highly luminous cA stars were used for supergiants of types B8–A5. The values of the average peculiar radial velocities θ were not used, the actual distribution being utilized. The radial-velocity distribution for B0–B5 stars was used for both the B3 and B5 groups of stars.

The well-known high value for the solar motion derived from the B stars and the small value from the A stars are clearly shown in Table I. The former is generally believed to be due to the fact that the apparently bright B stars form a local system having a systematic motion relative to ordinary bright stars; while the latter results, I think, from a preponderance of stars belonging to the generalized Ursa Major group.

The calculated distribution of absolute magnitudes may be affected by the tendency of the bright B stars to form moving groups. The non-uniform distribution along the galaxy is not disturbing, since the actual distribution of sin λ for the proper-motion stars is used in the determination of the distribution of the variable $y=\log|V_0+\epsilon/\sin\lambda|-1.6756$. When the dispersion in the peculiar velocities ϵ is small compared with V_0, the distribution of y itself has a small dispersion. The advantage of using this expression for y instead of $\log|V_0 \sin\lambda+\epsilon| + \text{Const.}$, which would be preferable in case the spread in ϵ were large compared with V_0, more than offsets the disadvantage of having to omit a large number of stars for which sin λ is smaller than 0.5.

The distribution of $y=\log|V_0+\epsilon/\sin\lambda|-1.6756$ is in Table II. The mean values of y appear at the bottom of each column and, as shown in previous publications, may be used for deriving mean absolute magnitudes.

Table III gives the distributions of $x=\log|v|+0.2\,m-\log\sin\lambda$. Observed and computed distributions are shown under the headings "O" and "C." For values of x less than -1.7, the frequencies were computed from the total number of stars below that limit.

The distributions $F(x)dx$ are shown graphically in Figures 1–6, the observed values as points joined by straight lines, the values computed from the distributions of absolute magnitudes and linear velocities by continuous curves.

In deriving the corrections $\Delta F(M)$, it was convenient in several cases to introduce the additional condition that the frequencies

TABLE II

y	$F(y)dy$					
	B0-B2	B0-B5	B8-B9	A0	A2-A5	cA
−3.5	.....	.....	.....	1	.....	1
3.4	.....	.....	.....	0	1	0
3.3	.....	.....	.....	0	0	0
3.2	.....	.....	.....	0	0	0
3.1	1	.....	.....	0	0	0
3.0	0	1	.....	1	1	1
2.9	0	0	.....	0	0	0
2.8	0	0	1	0	1	1
2.7	0	0	0	1	0	1
2.6	1	0	0	1	1	1
2.5	0	1	1	1	1	1
2.4	1	0	0	1	1	1
2.3	0	1	1	2	2	2
2.2	1	1	1	2	2	2
2.1	1	1	1	3	3	3
2.0	1	1	1	3	4	4
1.9	2	2	1	4	4	5
1.8	2	2	2	5	5	6
1.7	3	2	2	7	6	7
1.6	3	3	3	8	8	9
1.5	4	4	4	10	10	11
1.4	5	5	5	13	13	14
1.3	7	6	6	16	16	16
1.2	8	8	7	21	20	19
1.1	11	10	9	26	25	23
1.0	14	13	12	34	32	28
0.9	17	18	17	44	42	35
0.8	24	26	27	59	51	45
0.7	41	47	44	86	77	55
0.6	77	80	69	109	104	69
0.5	106	110	110	115	112	82
0.4	132	135	170	115	113	111
0.3	165	162	178	113	111	148
0.2	166	169	157	92	101	129
−0.1	99	115	100	68	70	82
0.0	56	55	47	30	41	49
+0.1	29	13	16	6	17	29
0.2	15	6	5	2	4	9
0.3	6	2	2	1	1	1
0.4	2	1	1	.....	.....	.....
+0.5						
$\bar{y}$.....	−0.339	−0.351	−0.352	−0.545	−0.521	−0.479

$F(M)dM$ cannot be negative. Between the main-sequence stars and the supergiants there is usually a region where the solution is some-

TABLE III

x	$F(x)dx$											
	B_0–B_2		B_3		B_5		B_8–B_9		A_0		A_2–A_5	
	O	C	O	C	O	C	O	C	O	C	O	C
−4.2												
4.1	.....	0.1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4.0	.....	0.1	.....	.....	.....	.....	0	0.1	.....	.....	.....	.....
3.9	1	0.1	.....	.....	.....	.....	0	0.0	.....	.....	0	0.1
3.8	0	0.1	.....	.....	.....	.....	0	0.1	.....	.....	0	0.0
3.7	0	0.2	0	0.1	0	0.1	0	0.0	.....	.....	0	0.1
3.6	0	0.2	0	0.1	0	0.1	0	0.1	0	0.1	0	0.1
3.5	0	0.2	1	0.1	1	0.1	0	0.1	0	0.1	0	0.1
3.4	1	0.3	0	0.1	0	0.1	1	0.1	0	0.2	1	0.1
3.3	0	0.3	0	0.2	0	0.1	0	0.1	0	0.2	0	0.2
3.2	1	0.4	0	0.2	0	0.1	0	0.2	1	0.3	0	0.3
3.1	0	0.6	0	0.3	0	0.2	0	0.2	0	0.3	0	0.3
3.0	1	0.7	1	0.3	1	0.2	0	0.2	0	0.3	0	0.4
2.9	1	0.7	0	0.3	0	0.3	1	0.4	0	0.4	1	0.4
2.8	1	0.9	0	0.3	1	0.4	0	0.6	0	0.5	0	0.5
2.7	2	1.2	1	0.5	0	0.5	1	0.7	1	0.6	1	0.7
2.6	2	1.6	1	0.7	1	0.7	0	0.9	0	0.8	0	0.9
2.5	3	2.0	1	1.0	1	0.9	1	1.1	1	1.0	1	1.1
2.4	4	2.7	1	1.2	1	1.0	1	1.3	0	1.3	1	1.3
2.3	4	3.7	2	1.5	2	1.3	1	1.8	1	1.5	2	1.5
2.2	5	5.2	2	1.9	2	1.5	1	2.2	1	1.9	2	2.0
2.1	8	7.0	3	2.4	3	2.1	2	2.6	2	2.4	2	2.6
2.0	9	9.2	3	2.9	4	2.5	3	3.0	2	3.0	3	3.2
1.9	10	11.7	4	3.6	4	3.0	4	3.5	2	3.8	3	4.0
1.8	14	13.8	5	4.9	5	3.8	5	4.4	3	4.6	4	5.0
1.7	18	15.3	7	6.6	7	5.0	6	5.4	3	5.7	6	6.0
1.6	17	17.0	5	8.8	4	6.6	5	6.8	0	7.2	1	7.4
1.5	22	19.5	11	12.2	4	9.1	2	8.7	7	9.2	7	8.9
1.4	7	24.9	13	17.5	8	13.4	10	10.9	27	11.4	8	11.1
1.3	40	35.5	24	24.1	16	19.3	10	13.5	9	13.8	15	13.8
1.2	45	52.7	53	31.3	25	26.5	14	16.7	14	16.1	11	17.5
1.1	84	74.9	37	39.2	33	34.8	34	21.0	17	18.5	19	22.0
1.0	62	98.8	37	49.0	16	44.8	25	26.3	14	21.8	29	27.5
0.9	157	118.0	45	61.6	66	58.3	22	31.6	16	26.2	44	33.1
0.8	173	127.5	77	76.0	90	73.7	58	36.9	41	33.0	35	37.8
0.7	74	118.0	117	89.3	107	87.9	48	44.7	37	42.8	47	41.6
0.6	111	90.2	91	104.3	107	103.4	63	58.9	54	57.1	46	45.2
0.5	19	57.7	109	118.8	82	119.6	73	84.7	97	76.1	48	52.9
0.4	53	35.3	136	121.4	139	125.1	107	117.3	85	93.7	75	64.6
0.3	30	21.7	114	99.2	131	106.1	136	134.9	117	104.2	54	77.5
0.2	10	14.0	56	64.8	74	73.6	146	130.7	137	107.4	86	87.8
−0.1	11	8.1	16	32.3	41	40.7	104	102.4	90	103.1	95	92.3
0.0	0	4.2	27	13.7	16	20.1	85	65.8	93	89.1	95	90.6
+0.1	0	2.0	0	4.7	0	8.2	17	34.6	68	67.7	73	80.8
0.2	0	1.0	0	1.8	0	3.3	14	15.3	28	41.3	82	64.9
0.3	0	0.5	0	0.6	8	1.1	0	6.0	26	19.8	55	44.6
0.4	0	0.2	0	0.2	0	0.3	0	2.2	3	7.8	36	26.2
0.5	.....	.....	.....	.....	0	0.1	0	0.7	3	2.5	6	12.9
+0.6	.....	.....	.....	.....	.....	.....	0	0.2	0	0.7	4	5.3

TABLE III—*Continued*

x	$F(x)dx$											
	B0–B2		B3		B5		B8–B9		A0		A2–A5	
	O	C	O	C	O	C	O	C	O	C	O	C
+0.6	.....	.....	.....	.....	.....	.....	0	0.1	0	0.2	2	1.9
0.7	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0	0.6
0.8	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0	0.2
0.9	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0	0.1
+1.0												

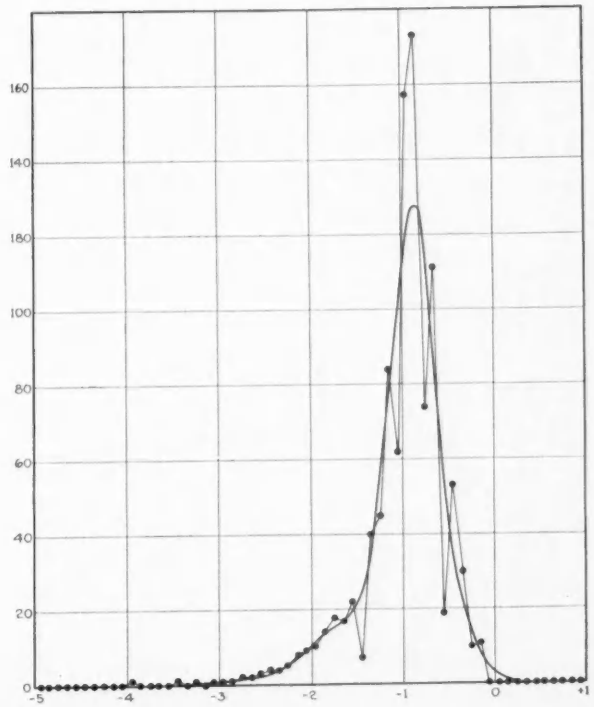

FIG. 1.—Distribution of $x = \log |v| + 0.2\, m - \log \sin \lambda$ for stars of spectral types B0–B2. The dots represent observed numbers within intervals $dx = 0.1$. The smooth curve is computed from the distributions of absolute magnitudes and of $y = \log |V_0 + \epsilon / \sin \lambda| - 1.6756$. For $x < -1.7$ the dots do not indicate independent observations, but show the probable extension of the distribution-curve.

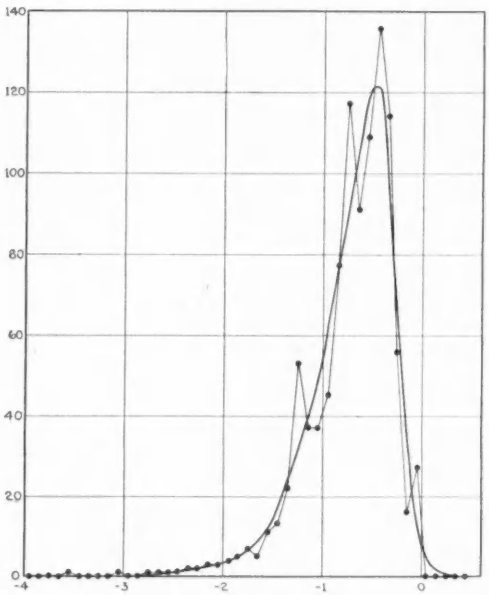

FIG. 2

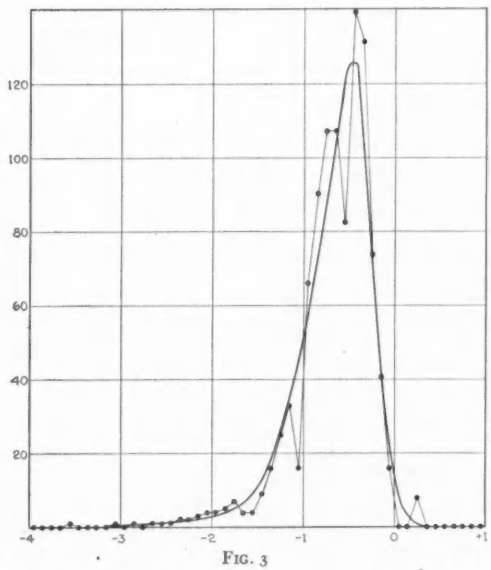

FIG. 3

FIGS. 2, 3.—Distributions of x for stars of spectral types $B3$ (above) and $B5$ (below). Otherwise similar to Fig. 1.

what uncertain. The values of $\Delta F(M)$ for this region were determined first from the least-squares solution; and if they proved to be negative and large enough to give negative values of $F(M)dM$, they were adjusted to give exactly zero frequencies for M. This was done to prevent the introduction of avoidable errors into adjacent parts of $F(M)dM$.

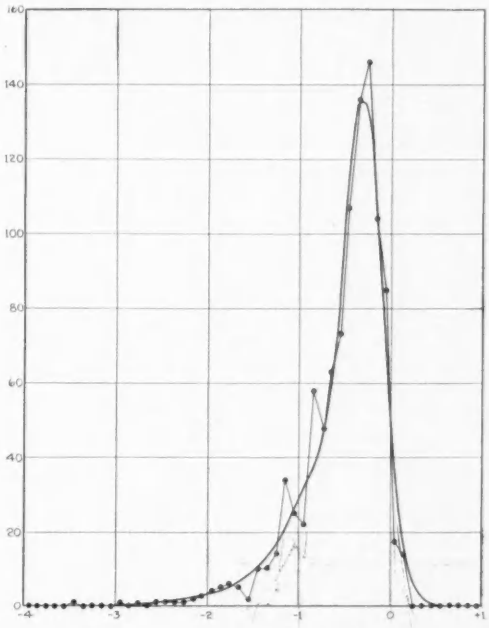

FIG. 4.—Distribution of x for stars of spectral types B8–B9. Otherwise similar to Fig. 1.

As in the previous studies, the intervals x_2-x_1, x_3-x_2, etc., and $M'_2-M'_1$, $M'_3-M'_2$, etc., have all been made equal to 0.4. Erratic fluctuations in $F(M)dM$ of small period have thus been eliminated, or at least greatly reduced.

The effect of the errors in the radial velocities and in the v-components was determined by the methods described in *Contribution* No. 395.[1] These effects are, in general, very small, and in practically all cases can be neglected.

The distributions of absolute magnitude are shown in Table IV

[1] *Astrophysical Journal*, **71**, 163, 1930.

and Figures 7–12. In order to follow the continuous change from later types, we will start with the stars of class A2–A5.[1] The lu-

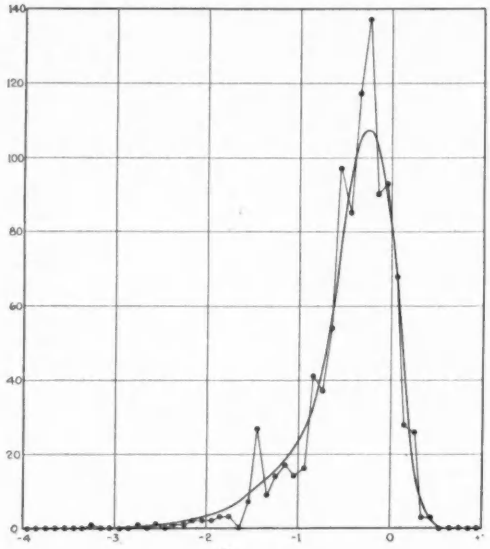

FIG. 5

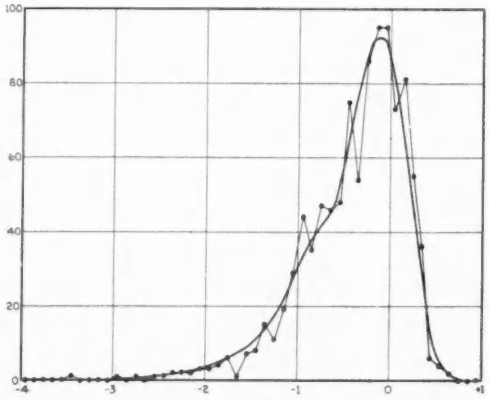

FIG. 6

FIGS. 5, 6.—Distributions of x for stars of spectral types A0 (above) and A2–A5 (below). Otherwise similar to Fig. 1.

minosity-curve shows two maxima: supergiants around $M = -3$ and a large group with maximum frequency at $M = +1.1$, including 15

[1] No A star is classified in *HD* as later than A5.

TABLE IV

M	$F(M)dM$					
	B0–B2	B3	B5	B8–B9	A0	A2–A5
−9.2	1	.....	.....	.....	.....	.....
9.0	3	.....	.....	.....	.....	.....
8.8	4	.....	.....	.....	.....	.....
8.6	5	.....	.....	.....	.....	.....
8.4	6	.....	.....	.....	.....	.....
8.2	6	.....	.....	.....	.....	.....
8.0	6	.....	.....	.....	.....	.....
7.8	5	.....	.....	.....	.....	.....
7.6	4	.....	.....	.....	.....	.....
7.4	3	.....	.....	.....	.....	.....
7.2	2	.....	.....	.....	.....	.....
7.0	0	.....	.....	.....	.....	.....
6.8	0	.....	.....	.....	.....	.....
6.6	0	1	.....	.....	1	.....
6.4	0	2	.....	.....	2	.....
6.2	0	4	.....	.....	3	.....
6.0	2	4	.....	.....	4	.....
5.8	5	3	.....	.....	4	.....
5.6	9	1	.....	.....	4	.....
5.4	6	0	.....	.....	4	.....
5.2	4	0	.....	.....	4	.....
5.0	0	5	1	1	3	.....
4.8	2	12	7	3	3	1
4.6	11	19	13	6	2	3
4.4	27	24	19	11	1	5
4.2	36	26	23	15	0	8
4.0	44	27	27	18	0	11
3.8	51	27	27	19	0	13
3.6	65	26	23	19	0	15
3.4	97	14	17	17	0	16
3.2	133	2	9	15	0	16
3.0	148	0	2	13	0	15
2.8	114	0	0	9	0	14
2.6	64	3	0	5	0	12
2.4	43	13	4	1	0	10
2.2	25	40	31	0	0	6
2.0	14	80	72	0	0	3
1.8	9	106	114	0	0	1
1.6	4	122	118	0	1	0
1.4	1	122	118	2	2	0
1.2	0	107	101	20	3	0
1.0	0	70	74	90	5	0
0.8	4	55	57	120	8	0
0.6	9	41	48	124	12	0
0.4	11	23	38	124	22	1
−0.2	8	13	25	116	58	8
0.0	5	6	12	90	109	26
+0.2	3	2	8	62	125	52
0.4	1	.....	5	38	129	80
0.6	.....	.....	4	26	129	93
0.8	.....	.....	2	17	120	96
+1.0						

TABLE IV—*Continued*

M	*F*(*M*)*dM*					
	B0–B2	B3	B5	B8–B9	A0	A2–A5
+1.0	.....	.....	1	11	96	97
1.2	.....	.....	.....	6	66	95
1.4	.....	.....	.....	2	32	88
1.6	.....	.....	.....	.....	21	72
1.8	.....	.....	.....	.....	14	58
2.0	.....	.....	.....	.....	8	40
2.2	.....	.....	.....	.....	4	18
2.4	.....	.....	.....	.....	1	10
2.6	.....	.....	.....	.....	.....	5
2.8	.....	.....	.....	.....	.....	3
3.0	.....	.....	.....	.....	.....	3
3.2	.....	.....	.....	.....	.....	2
3.4	.....	.....	.....	.....	.....	2
3.6	.....	.....	.....	.....	.....	1
3.8	.....	.....	.....	.....	.....	1
+4.0						

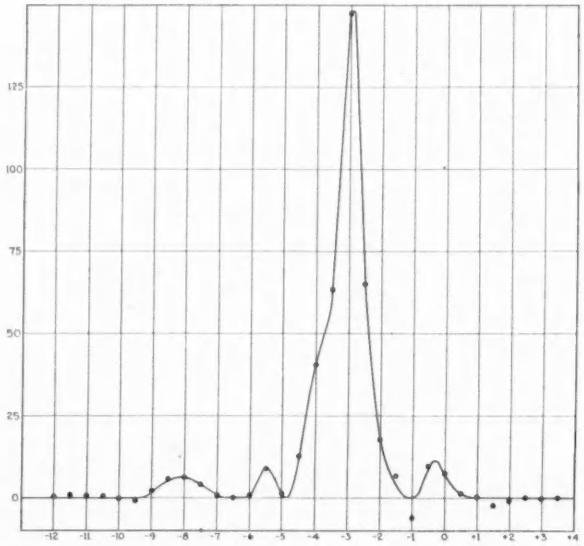

FIG. 7.—Distribution of absolute magnitudes for stars of spectral types B0–B2 brighter than the sixth apparent magnitude.

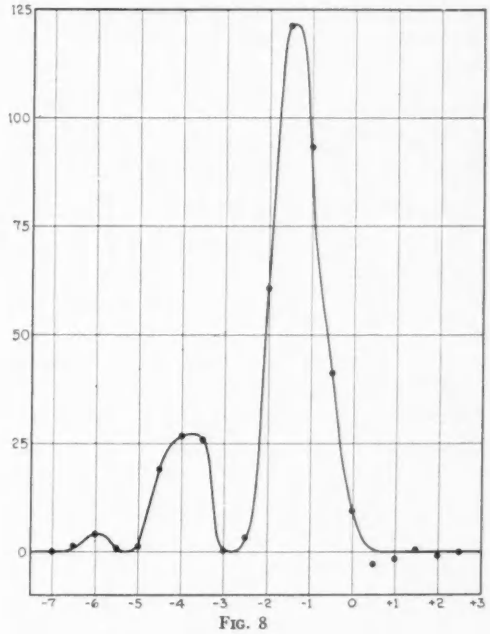

FIG. 8

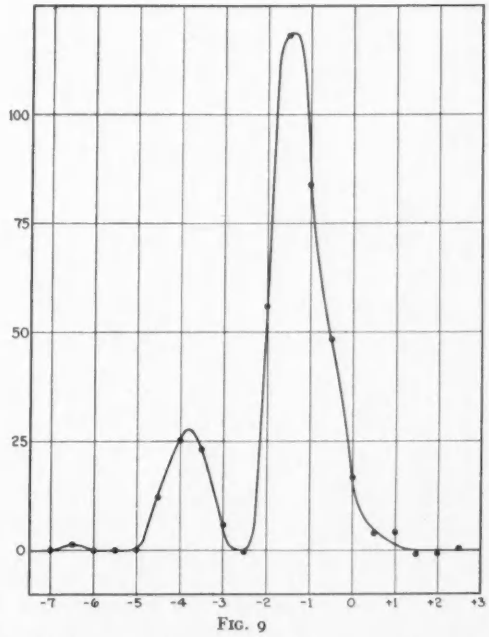

FIG. 9

FIGS. 8, 9.—Distributions of absolute magnitudes for stars of spectral types B3 (above) and B5 (below) brighter than the sixth apparent magnitude.

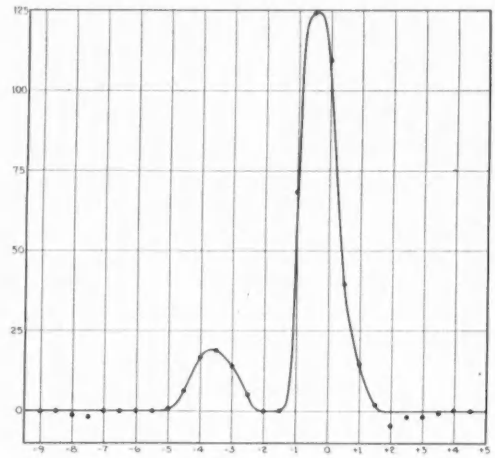

FIG. 10

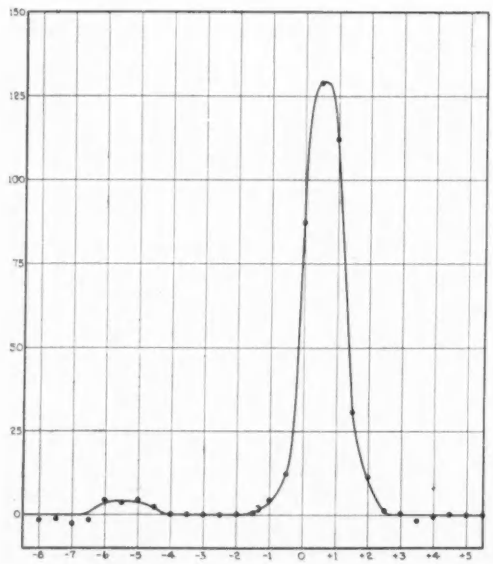

FIG. 11

FIGS. 10, 11.—Distributions of absolute magnitudes for stars of spectral types B8–B9 (above) and A0 (below) brighter than the sixth apparent magnitude.

and 85 per cent, respectively. Comparing this curve with that of the F stars,[1] we see that the dwarfs have increased in intrinsic brightness from +3.2 to around 1 or 2 and have coalesced with the group of "normal giants." There may still exist differences between the giants and the dwarfs, depending on criteria other than intrinsic brightness and density; but we have, as yet, no evidence of such a distinction. Since we have no right to suppose that the principal group consists of dwarfs or main-sequence stars in preference to normal giants, we

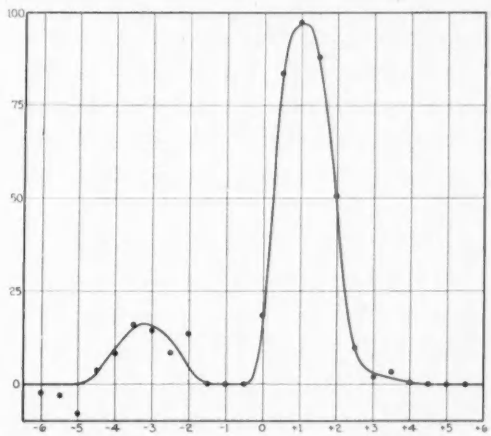

FIG. 12.—Distribution of absolute magnitudes for stars of spectral types A2–A5 brighter than the sixth apparent magnitude.

will designate this group as "normal stars" of type A2–A5, and regard it as a mixture of main-sequence stars and normal giants.

Among the A0 stars is a group having the mean absolute magnitude −5.5, two magnitudes brighter than the supergiants of late A type. The normal A0 stars have a mean absolute magnitude of +0.7. The relative numbers of stars in the two groups are 4 and 96 per cent, respectively. The existence of the first group might well be questioned, were it not for the fact that we know definitely a few exceedingly bright stars, like α Cygni, of spectral type A. This group probably consists exclusively of c stars, whereas supergiants of other types must include many besides c stars, as may be judged from the relatively large percentages involved.

[1] *Mt. Wilson Contr.*, No. 430; *Astrophysical Journal*, 74, 110, 1931.

The distribution-curve for the stars of class B8–B9 shows supergiants with a mean absolute magnitude -3.5 and normal B8–B9 stars around $M = -0.3$, in the proportion 15 and 85 per cent. The group of normal stars shows the beginning of an asymmetry which indicates the existence of a group of stars of mean absolute magnitude about $+1$. This hidden group, the evidence for which will be strengthened by the study of earlier types, may be a remnant of normal giants, since the mean absolute magnitude of these stars does not change much from one spectral class to another and for the F stars has the value $+1.3$. If this identification is justified, we may also say that the main-sequence stars (dwarfs) are now brighter than the normal giants; in other words, that the main-sequence branch and the line of normal giants cross one another in the A stars. For A and B stars we will henceforth drop the term "dwarf" as somewhat misleading.

Among the B5 stars the supergiants have a mean absolute magnitude -3.8, and the normal stars a maximum frequency at $M = -1.4$, the proportions being 17 and 83 per cent, respectively. The normal giants appear as a bulge at an absolute magnitude of about 0.0. There is also a slight indication of a small group at $M = -6.5$, consisting of 0.5 per cent of the total numbers. This group is not included in the tables, but is indicated in Figure 9.

The B3 stars show two groups of supergiants. The first group, which is somewhat uncertain, has a maximum at -6, the second at -4. The normal stars have a maximum frequency at -1.4. The proportions of stars in the three groups are 2, 18, and 80 per cent, respectively. The first group, as judged from their number and in analogy with the A0 stars, may be the regular c stars. The frequency-curve for the normal star is asymmetrical and indicates a submerged group of normal giants around $M = -0.3$.

The group of B0–B2 stars is the most difficult to analyze on account of the small number of stars involved and the smallness of the proper motions. The analysis gives four distinct groups, with maxima in the luminosity-curve at -8, -5.5, -2.9, and -0.3, and relative proportions of 4.5, 2.5, 89, and 4 per cent. There is also a bulge clearly indicating a submerged group at -4. If we compare with the B3 stars, we can recognize the two groups of supergiants at -5.5

and -4, the main-sequence stars at -2.9, and the normal giants at -0.3, now well separated from the main-sequence stars. The group at $M=-5.5$ may be the regular c stars, as suggested in the case of the B_3 stars.

The new group appearing at absolute magnitude -8 will be of great interest, if its reality can be established. The evidence—or rather lack of evidence—may be judged from the following consideration. Among the 124 stars of types Bo–B_2, we have, by the analysis, found 5.6 stars that have this extreme brightness. Their existence is derived mainly from the fact that x is less than -1.7 for 7.5 stars, whereas we should expect only 3.9 such stars if these highly luminous stars were absent. For mean values of m and of sin λ, the foregoing limit of x corresponds approximately to $v=0''.003$. The determination thus depends mainly on an observed excess of about 4 stars having v-components less than $0''.003$.

In general, the part of the luminosity-curve that represents the distribution of fainter stars can be determined with much higher accuracy than the part for the brightest stars.

It is not easy to express the probability of the reality of the different groups by any definite figures. For late-type stars we may judge the evidence by the concordance of the results obtained from parallactic and peculiar motions, and, in addition, by the general run from type to type. For the earlier types we can use only parallactic motions, or motions parallel to the galaxy; and we may best judge the reality of the different groups by comparing the results from different spectral intervals. It is to be noted that among the A and B stars there is no overlapping of observational material; each spectral group is analyzed quite independently of other groups.

The next group in the spectral sequence to be considered would be the late O-type stars. These stars are too few to be analyzed separately by the present method, and they cannot be included among the early B stars since their radial velocities are in the average larger than those of the apparently bright B stars. J. S. Plaskett's[1] work on the O5–O9 stars indicates a mean absolute magnitude of about -3.7 for stars down to apparent magnitude 7.7. With allowance for the progressive change in the main-sequence group, this is about

[1] *Publications of the Dominion Astrophysical Observatory*, 2 (No. 16), 325, 1924.

what would be expected from a slight extrapolation from the Bo–B2 stars. Plaskett also finds that the apparently faint O stars have an absolute magnitude of −0.45, which would put them in the group here designated as "normal giants."

The present results may be affected by systematic errors in the proper motions used, as for instance, by those due to rotation terms. The correction of such errors would in general tend to increase the number of very small proper motions, and hence make the brightest stars somewhat brighter. The absorption of light in the galaxy would also cause a tendency to underestimate the intrinsic luminosity of the brightest stars. Some of the groups found in this study are unexpectedly luminous; but we must remember that we have hitherto always derived mean absolute magnitudes for selected groups of stars; and if our criteria for selecting extremely bright objects within the galaxy are not very sharply defined, there will be a tendency to make the mean absolute magnitudes too faint.

A summary and discussion of the luminosity-curves for spectral types Bo–M will appear soon in another *Contribution*.

The writer wishes to express his thanks to Miss Myrtle Richmond and Miss Elizabeth MacCormack for their assistance with much of the numerical work involved in this study.

Carnegie Institution of Washington
Mount Wilson Observatory
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